KSCE Journal of Civil Engineering (2012) 16(1):107-117
DOI 10.1007/s12205-012-1152-1

Structural Engineering
www.springer.com/12205

Concept and Analysis of Stay Cables with a CFRP and Steel Composite Section

Wen Xiong*, C.S. Cai, Rucheng Xiao***, and Lu Deng******

Received February 26, 2010/Revised 1st: August 1, 2010, 2nd: April 1, 2011/Accepted April 8, 2011

Abstract

This paper describes the concept of a novel stay cable with a Carbon Fiber Reinforced Plastic (CFRP) and steel composite section. In this concept CFRP core conserves all the advantages of CFRP materials such as light weight and high strength, while steel coat provides protection for the CFRP core and ensures reliable anchorage performance. The steel coat can reduce cost and provide higher stiffness than CFRP materials in some length range. Following this concept, several configurations were proposed for the composite structure. Effects of the key design parameter of the proposed stay cables, namely the ratio of the CFRP section area to the whole section area, were examined through a parametric study using analytical solutions. By doing this, an appropriate range of the ρ value was suggested. Factors that can affect the appropriate range of the ρ value, including the horizontal projected length of stay cables, cable force, and pylon height (i.e., vertically projected length of stay cables), were also studied. Finally, the feasibility of the proposed stay cables was conceptually verified. It was shown that the proposed stay cables could be an excellent alternative to the pure CFRP or traditional steel stay cables.

Keywords: *stay cable, Carbon Fiber Reinforced Plastic (CFRP) and steel composite section, CFRP core, steel coat, cable-stayed bridge, parametric analysis*

1. Introduction

In recent years, there has been extensive research on the application of Carbon Fiber Reinforced Plastic (CFRP) stay cables in bridge engineering (Meier, 1987; Saadatmanesh and Ehsani, 1998; Schurter and Meier, 1996; Mei *et al.*, 2005). As a new material for stay cables, the light weight and superior strength of the CFRP can be utilized to improve the load-carrying efficiency of stay cables and extend the span length of cable-stayed bridges to over 1000 m.

One of the drawbacks with CFRP materials is their brittle performance. CFRP materials do not resist shearing and transverse compressions as good as steel materials do. Therefore, the anchorage of CFRP stay cables needs to be specially designed to avoid the failure of stay cables by transverse shear. Actually the anchorage problem has recently attracted the attention of many researchers (Burtscher, 2008; Sayed-Ahmed, 2002); however, few papers have presented a satisfactory and acceptable anchorage solution for field applications. Besides, the high material and manufacture cost of CFRP remains an issue for its wide applications. Moreover, the relatively low elastic modulus of CFRP materials also causes a low axial stiffness of stay cables which can decrease the overall stiffness of the cable-stayed bridges. Because of all these issues, until currently CFRP stay cables still cannot compete with the traditional steel stay cables. To fully utilize the advantages of CFRP materials in the application of stay cables, a good strategy of using a CFRP and steel composite section is desirable.

To address this issue, this paper proposes a novel stay cable with a CFRP and steel composite section. The strands of the proposed stay cables are fabricated by wrapping a steel coat around a CFRP core to form a composite section. This is to create a new structure that keeps only the advantages of each of the two constituents. More specifically, CFRP core can act as a load carrying component and conserve all the advantages of CFRP materials such as their light weight and high strength, while the steel coat can be seen as a protection medium for CFRP core to relieve the high local stress acting on CFRP materials. With this design, the anchorage problem can be theoretically avoided by anchoring stay cables on the steel coat. The steel coat is also a load carrying component, in addition to being a protection medium, which increases the axial stiffness of stay cables and reduces the total cost of the materials for a certain range of cable lengths. It is easy to understand that the performance of the composite stay cables

*Assistant Professor, Dept. of Bridge Engineering, School of Transportation, Southeast University, Nanjing 210096, China (E-mail: wxiong12@gmail.com)
**Professor, Dept. of Civil and Environmental Engineering, Louisiana State University, Baton Rouge, LA 70803, USA (Corresponding Author, E-mail: cscai@lsu.edu)
***Professor, Dept. of Bridge Engineering, Tongji University, Shanghai, 200092, China (E-mail: xiaorc@tongji.edu.cn)
****Professor, College of Civil Engineering, Hunan University, Changsha, 410082, China (E-mail: ludeng9@gmail.com)

is determined by both CFRP and steel materials. Due to the different contributions from the two materials, the material proportion of the composite section is regarded as a principal and basic factor to affect the stay cable behaviors and cost. For this reason, in the present study the ratio of the CFRP section area to the whole section area $\rho = A_{CFRP}/A_{STEEL+CFRP}$ is determined as the key design parameter for the composite section. By looking for an appropriate area ratio, the advantages of both CFRP and steel materials can be fully and purposely utilized in the composite stay cables.

In this study, several configurations of the composite structure were firstly proposed. The area ratio ρ was examined through a parametric study using analytical solutions, based on which an appropriate range of ρ values was suggested. Factors that can affect the appropriate range of ρ values, including the horizontally projected length of stay cables, cable force, and pylon height (i.e., vertically projected length of stay cables), were also studied. Finally, the feasibility of the proposed stay cables was conceptually verified in two ways: the possibility of fully utilizing the total strengths of the two different materials under ultimate loads and redistribution of the internal force between both materials under the temperature-induced loads. It was shown that the proposed stay cables could be an excellent alternative to the pure CFRP or traditional steel stay cables, especially for super long-span bridges.

2. Proposed CFRP and Steel Composite Section

A strand of the proposed stay cables is shown in Fig. 1 where the composite section is formed by wrapping a steel coat around the single CFRP core, namely single-strand. The steel coat can be made of a hollow circle (Fig. 1a) or several wires (Fig. 1b, which is preferred). In this configuration, both CFRP and steel materials can fully utilize their total strengths in the ultimate condition (see explanation later). Before reaching the ultimate condition, the bond between the steel coat and CFRP core, especially in the anchorage zone, plays a critical role in that it makes the two materials work as an integrated system by preventing sliding between the two materials when subject to external loads. As a matter of fact, many studies have been conducted and reported on the adhesive between the CFRP laminates and steel girder for the structural strengthening applications (Sen *et al.*, 2001; Matta, 2003; Schnerch *et al.*, 2004; Colombi and Poggi, 2006; Rizkalla *et al.*, 2008; Zhao, 2008). Zhao and Zhang (2007) also gave a state-of-the-art review on CFRP strengthened steel structures including bond test method, failure modes in CFRP bonded steel system, bond strength, and bond-slip relationship. All the studies proved that appropriate bond materials/techniques between steel and CFRP materials do exist and can be reliably applied in the strengthening applications. By using the same idea, it is reasonable to believe that a reliable bond between the steel coat and CFRP core in this composite stay cable design could be obtained by using the same or similar bond materials/techniques used for structural strengthening. The bond issue should not be an obstacle for the design of composite stay cables.

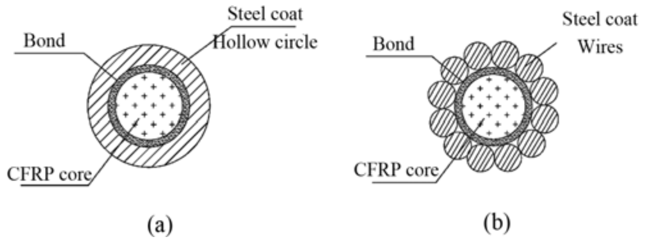

Fig. 1. Proposed Single-strand Composite Section: (a) Steel Coat (Hollow circle), (b) Steel Coat (Wires)

Similar to the assembly of steel stay cables, the single composite strands shown in Fig. 1 can be also assembled into one stay cable, as shown in Fig. 2. While the CFRP core conserves the light weight and superior strength of CFRP materials, the steel coat makes it possible to assemble the composite strands and anchor the proposed stay cables using existing post-tensing and anchorage techniques for steel stay cables. The anchorage design for the composite stay cables (strands with composite sections) are shown in Fig. 3.

The composite section or bond area in the composite structure can also be limited to the anchorage zone of stay cables where the steel coating is purely for conveniently anchoring the CFRP core/wires (see Fig. 4a). However, in this case an enough transfer length of the steel coat (hollow circle or wires) needs to be carefully determined, which will assure a complete transfer of anchorage force from the anchorage device to the CFRP core/wires. Experiments on the transfer length are desired and further verified by theoretical analysis. In other non-anchorage areas there could be no bond requirement between the steel coat and CFRP core under the external loads. This study focuses on the case of a composite section along the whole cable length as shown in Fig. 4(b).

Figure 5 presents another configuration of the composite strand where the core consists of several CFRP strands, namely multi-strands, as compared to a single strand in Fig. 1.

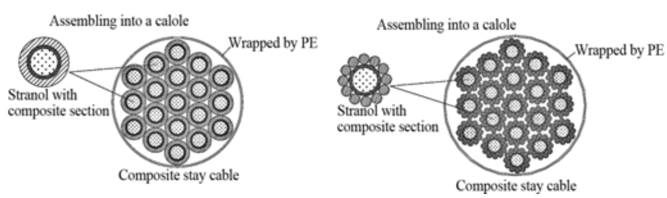

Fig. 2. Assembled Single Strands

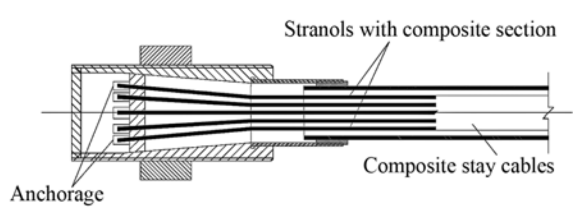

Fig. 3. Anchorage Design for Composite Stay Cables

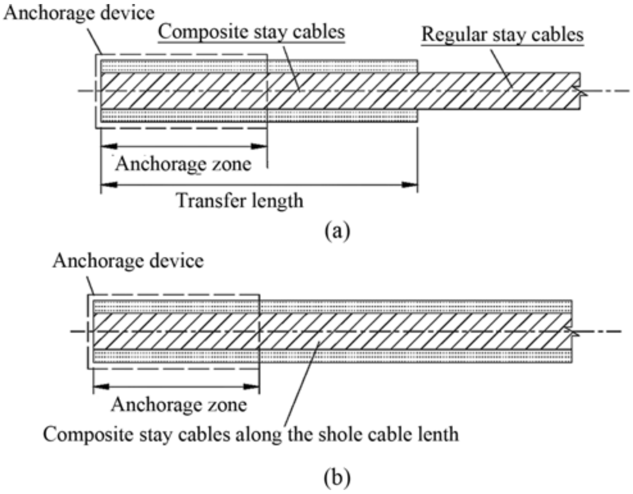

Fig. 4. Application Range of Composite Structure: (a) Composite Section at the Cable End Only, (b) Composite Section for the Entire Cable

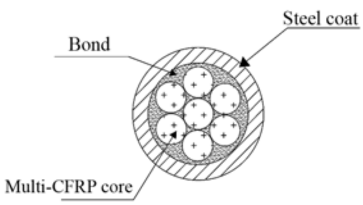

Fig. 5. Proposed Multi-strand Composite Section

3. Analytical Solutions of Cable Elements

No matter which configuration is used, the area ratio ρ discussed earlier is the key design parameter. Before conducting the parametric study of the area ratio ρ, analytical solutions for cable shape was firstly derived below in detail. Expressions for the five other parameters of stay cables, including cable sag effect, equivalent elasticity modulus of cable elements, load-carrying efficiency of stay cables, stress analysis of stay cables under self-weight, and cable geometric-elasticity parameter λ^2, are also introduced next. The obtained analytical solutions discussed below will serve the basis for the parametric studies that will be described later.

3.1 Cable Shape Analysis

In the following analysis, it is assumed that: (1) the cable elements are perfectly flexible, i.e., the cable's ability to resist the compression and bending is ignored (Peyrot and Goulois, 1978; Jayaraman and Knudson, 1981); (2) linear elasticity is applicable to the cable materials (Peyrot and Goulois, 1978; Jayaraman and Knudson, 1981).

Considering an elastic cable element stretched in a vertical plane as shown in Fig. 6, the equilibrium shape under the uniform gravity load (self-weight) can be described with an elastic catenary. The differential equation of the cable shape (Eq. (1)) has been obtained by many researchers (Irvine, 1981; Ahmadi-Kashani and Bell, 1988; O'Brien and Francis, 1964):

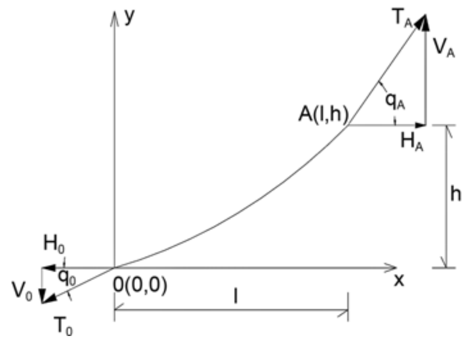

Fig. 6. Coordinate System and Sign Convention

$$y = \int y'dx = \frac{H}{q_0}\left(\sqrt{1+y'^2} + \frac{H}{2K}y'^2\right) + c_1$$
$$= \frac{H}{q_0}\sqrt{1+y'^2}\left(1 + \frac{H}{2K}\sqrt{1+y'^2}\right) + c'_1 \quad (1)$$

where H = horizontal components of the cable forces; q_0 = distributed loads (uniform gravity loads) per unit length of the unstressed cable element; $K = E \cdot A_0$; E = elasticity modulus; A_0 = area of the cross section of the unstressed cable element; and c_1 and c_1' = any arbitrary constant.

3.2 Cable Sag Effect

The equivalent stiffness of cable elements decreases with the increase of sag and eventually leads to a reduction of load-carrying efficiency of stay cables (Freire *et al.*, 2006). The cable sag effect, commonly referred to as the ratio of the axial deformation δ to the chord length L_c of cable elements (Gimsing, 1997), must be taken into account, especially in the analysis of long-span cable-stayed bridges. The derivation of the analytical solution of the cable sag effect is briefly shown below. The analytical solution of the cable sag effect can be written as (Gimsing, 1997):

$$\frac{\delta}{L_c} = \frac{(\sigma_2 - \sigma_1)\gamma_c + \frac{1}{L_c}\left[\sigma_2^2 sh\left(\frac{\gamma_c l}{\sigma_2}\right) - \sigma_1^2 sh\left(\frac{\gamma_c l}{s_1}\right)\right] - \frac{4E}{L_c}\left[\sigma_2 sh\left(\frac{\gamma_c l}{2s_2}\right) - s_1 sh\left(\frac{\gamma_c l}{2s_1}\right)\right]}{2g_c E\,\mathrm{ch}\left(\frac{\gamma_c l}{2\sigma_2}\right)} \quad (2)$$

where δ/L_c = cable sag effect; L_c = chord length of the cable element; l = horizontal projected length of the cable element; σ_1 and σ_2 = stress of the cable element in two load conditions, respectively; and $\gamma_c = q_0/A_0$ = density of the cable element.

3.3 Equivalent Elasticity Modulus of Cable Elements

The concept of equivalent elasticity modulus of cable elements was first introduced by Ernst (1965). It is usually assumed that if the change in tension of cable elements during a load increment is not significant compared with the existing tensioned cable forces, the axial stiffness of cable elements will not change significantly and the equivalent elasticity modulus of cable elements E_{eq} can be considered constant during the load increment. E_{eq} can be given by the following equation (Gimsing

1997):

$$E_{eq} = \frac{1}{\frac{1}{E}+\frac{\gamma_c^2 l^2}{24}\left(\frac{\sigma_1+\sigma_2}{\sigma_1^2\sigma_2^2}\right)} \approx \frac{E}{1+\frac{\gamma_c^2 l^2}{12\sigma_1^3}E} \tag{3}$$

3.4 Load-carrying Efficiency of Cable Elements

In the present study, the load-carrying efficiency of cable elements is referred to as the ratio of the vertical component of the cable force in the actual curved cable to that in the numerically simulated cable using a single straight truss element. The main difference between the actual curved cable and the simulated cable is whether or not to consider the cable sag effect when calculating the cable forces.

The load-carrying efficiency of cable elements can be expressed as:

$$\frac{V_A}{V_0} = \frac{\left(-\frac{l}{l_{max}}+1\right)\sqrt{1+k^2}}{\sqrt{1+k^2\left(-\frac{l}{l_{max}}+1\right)^2}} \tag{4}$$

where V_A = vertical component of the cable force in the actual curved cable; V_0 = vertical component of the cable force in the numerically simulated cable using a single straight truss element; $k = h/l = h_{max}/l_{max}$; h = vertically projected length of the cable element; and l_{max}= ultimate horizontally projected length of the cable element, which can be solved from the following equation (Gimsing, 1997):

$$\frac{[\sigma]/\gamma_c}{[\sigma]/\gamma_c - k\cdot l_{max}} = \mathrm{ch}\left(\frac{l_{max}}{[\sigma]/\gamma_c - k\cdot l_{max}}\right) \tag{5}$$

where $[\sigma]$ = tensile strength of the cable element.

3.5 Stress Analysis of Cable Elements Under Self-weight

The cable stress caused by the self-weight of cable elements, given by the equation shown below (Gimsing, 1997), is also an important parameter. A low value of this parameter reflects a high capacity of cable elements to carry extra external loads.

$$\sigma_g = \frac{\gamma_c l\sqrt{1+k^2}\sqrt{1+k^2(1+l/l_{max})^2}}{2k(1+l/l_{max})} \tag{6}$$

3.6 Cable Geometric-elasticity Parameter λ^2

Cable geometry-elasticity parameter (λ^2) is referred as to the ratio of the axial stiffness to the geometry stiffness of cable elements (Cai *et al.*, 2006). This parameter represents the characteristic of the in-plane symmetrical vibration of cable elements, which can be written as (Cai *et al.*, 2006):

$$\lambda^2 = \frac{(q\cos\alpha L_c/H)^2 L_c}{(HL_e/EA)} = \frac{(\gamma\cos\alpha)^2 EL_c^3}{(\sigma^3 L_e)} \tag{7}$$

where $L_e = L_c[1+8(f/L_c)^2]$; $f = (q\cos\alpha L_c^2)/8H$; $\gamma = q/A$; α = inclination angle of the cable element; and $\sigma = H/A$. All the discussed parameters that describe the characteristics of cable elements will be used in the following parametric studies.

4. Key Parameter Design

As discussed earlier, the area ratio ρ is determined as the key design parameter for the composite strand of the proposed stay cables. To better understand the effects of ρ on the performance of the proposed stay cables, in the following section seven mechanical parameters (see Table 2) were theoretically investigated through a parametric study. Based on the results of the parametric study an appropriate range of ρ values can be recommended, which can assist engineers to perform and optimize their designs and fully utilize the advantages of both steel and CFRP materials.

4.1 Analytical Models of Stay Cables

To cover more possible configurations of stay cables, four analytical models of the stay cables with different horizontally projected lengths, 340 m, 700 m, 1,400 m, and 2,500 m, respectively, were considered. A summary of some details of the four analytical models, including the geometry and material properties, is shown in Table 1. Using the four analytical models, seven mechanical parameters (see Table 2) were investigated in detail one by one by varying the variable ρ from 0% to 100%. These seven mechanical parameters can comprehensively describe the characteristics of cable elements, and all of them are the key study objectives for the cable-related research (Irvine, 1981; Ahmadi-Kashani and Bell, 1988; O'Brien and Francis, 1964; Freire *et al.*, 2006; Gimsing, 1997). All the equation numbers used to solve the seven investigated mechanical parameters are also listed in Table 2.

It should be noted that the geometrical configurations and physical properties of the four analytical cable models were taken directly or referred partially from the preliminary designs of two steel cable-stayed bridges with a 1,400 m main span (Nagai *et al.*, 2004; Lin, 2004; Miao, 2006). Also, based on self-weight of the girder in the two preliminary designs, the cable forces used in

Table 1. Parameters of Four Stay Cable Models

	CFRP core	Steel coat
Chord length of stay cables (m)	415.2, 754.6, 1507.8, 2692.6	
Horizontal projected length of stay cables (m)	340, 700, 1400, 2500	
Vertical projected length of stay cables (m)	238.4, 281.8, 560,1000	
Density (kg/m^3)	1600	7850
Area (m^2)	0.010	
Elasticity modulus (kN/m^2)	1.37E+08	2.00E+08
Axial thermal Coefficient of expansion (E-6/°C)	0.68	11.7
Transverse thermal Coefficient of expansion (E-6/°C)	25	11.7
Tensile strength (MPa)	2600	1860
Safety factor	2.5	2.5
Cable force (kN)	6.80E+6	

Table 2. Investigated Objectives

Item	Investigated Objectives	Eq.
a*.	Total axial deformation of stay cables	(1)
b*.	Sag at the middle span of stay cables	(2)
c.	Load-carrying efficiency of stay cables	(4)
d.	Stress analysis of stay cables under self-weight	(6)
e.	Cable geometric-elasticity parameter λ^2	(7)
f.	Equivalent elasticity modulus of stay cables	(3)
g.	Cable sag effect	(2)

Note: *Objectives "a" and "b" need to be obtained by further derivation using the equations.

the four analytical cable models can be proximately set as 6.80E+6 kN with a safety factor of 2.5, which were slightly higher than a typical cable force due to the large span of the cable models. In order to rationally compare the results from the parametric study, the cable forces and section area in all the cases were kept the same.

4.2 Results of Analysis

Figs. 7(a~g) show the performance of the stay cables with regarding to the investigated mechanical parameters by varying the ρ value from 0% to 100%. Firstly, similarity can be observed in Figs. 7(a~e) where the four curves in each figure for different cable models show the same trend as the area ratio ρ increasing from 0% to 100%. Moreover, when the ρ is close to 100%, CFRP materials show dominating effects on the investigated mechanical parameters. However, it is noted that in this case the small amount of steel materials cannot ensure a good anchorage performance. On the other hand, if a high area ratio ρ is employed, which means more CFRP area in the composite section, the high material and manufacture cost of CFRP will be an issue. As a result, an appropriate area ratio ρ should be selected with the overall consideration of mechanical behavior, constructional (anchorage) performance, and material and manufacture cost. The constructional performance refers to the anchorage performance and the economical performance refers to the material and manufacture cost of CFRP hereafter unless specifically noted otherwise.

Meanwhile, as shown in Figs. 7(f) and 7(g), a high area ratio ρ (more CFRP materials) does not necessarily guarantee excellent performance of stay cables in terms of the concerned mechanical parameters. In Fig. 7(f), i.e., the plots of the equivalent elasticity modulus of stay cables, not all curves have the same pattern. For example, the curve of the 1400 m-span cable model rises when the area ratio ρ changes from 0.0 to 0.6 and declines thereafter. The cable model with a span of 2,500 m, however, presents a continuous upward trend in the entire range of ρ values, which is also quite distinct from the cases of the 340m span and 700 m span models. It should be noted that a high equivalent elasticity modulus of stay cables is usually preferred because it is the main contributor to the overall stiffness of the cable-stayed bridges. Since the maximum equivalent elasticity modulus of stay cables corresponds to different ρ values in different curves, the appropriate ranges of ρ value for the cable models with spans of 340 m, 700 m, 1,400 m, and 2,500 m, should be set as close to 0.0, 0.0, 0.6, and 1.0, respectively. Similar behaviors are also observed in the case of cable sag effect in Fig. 7(g) where the four analytical models of stay cable exhibit a significant difference. Based on

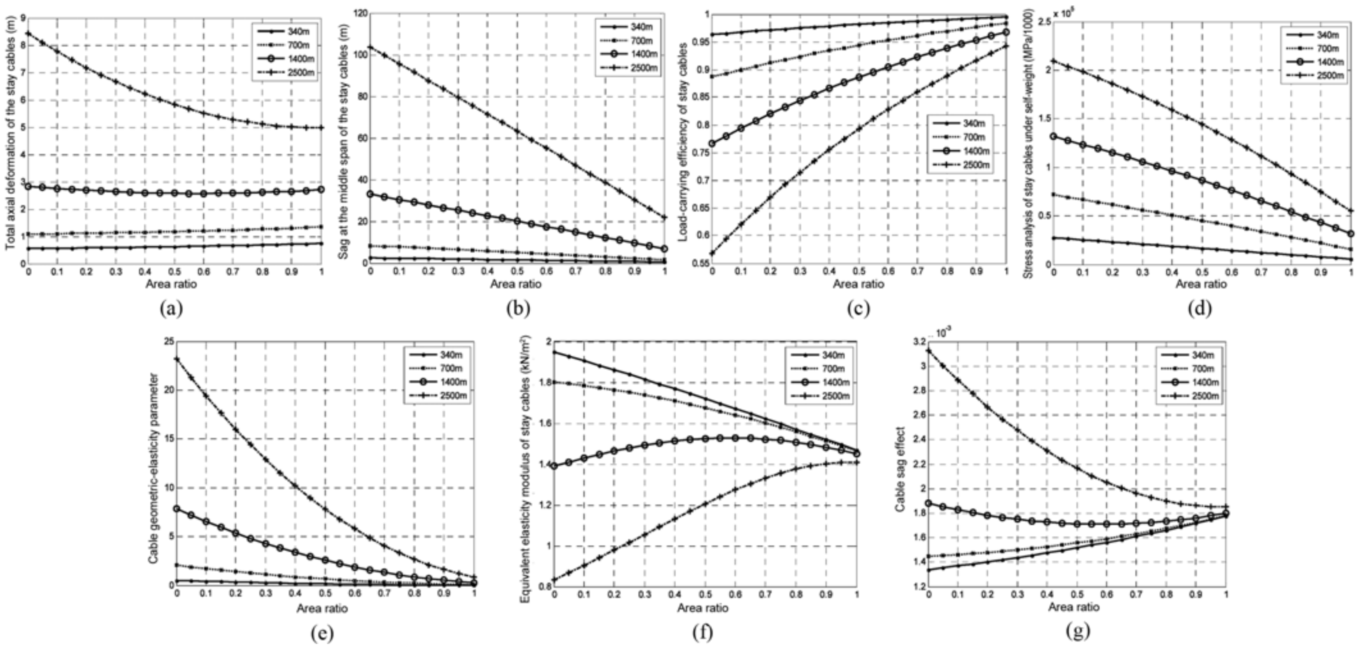

Fig. 7. Results of Parametric Study: (a) Total Axial Deformation of Stay Cables, (b) Sag at the Middle Span of Stay Cables, (c) Load-carrying Efficiency of Stay Cables, (d) Stress Analysis of Stay Cables Under Self-weight, (e) Cable Geometric-elasticity Parameter λ^2, (f) Equivalent Elasticity Modulus of stay Cables, (g) Cable Sag Effect

Table 3. Appropriate Area Ratio ρ

Mechanic Behaviors	Criterion	Appropriate ranges			
		340m	700 m	1400 m	2500 m
Total axial deformation of stay cables	The lower is preferred	Any value	Any value	Any value	Close to 1.0
Sag at the middle span of stay cables	The lower is preferred	Any value	Close to 1.0	Close to 1.0	Close to 1.0
Load-carrying efficiency of stay cables	The higher is preferred	Any value	Close to 1.0	Close to 1.0	Close to 1.0
Stress analysis of stay cables under self-weight	The lower is preferred	Close to 1.0	Close to 1.0	Close to 1.0	Close to 1.0
Cable geometric-elasticity parameter λ^2	The lower is preferred	Close to 1.0	Close to 1.0	Close to 1.0	Close to 1.0
Equivalent elasticity modulus of stay cables	The higher is preferred	Close to 0.0	Close to 0.0	0.45~0.85	0.8~1.0
Cable sag effect	The lower is preferred	Close to 0.0	Close to 0.0	0.55~0.75	0.9~1.0

Note: Good constructional and economical performance always require a low area ratio ρ.

these observations, the selection of the appropriate area ratio ρ obtained based on the Figs. 7(f) and 7(g), i.e., equivalent elasticity modulus and cable sag effect, should be different from that based on the results in Figs. 7(a~e).

In summary, using only one specific ρ value cannot guarantee satisfactory results for all the investigated mechanical parameters. It becomes more complicated if both the constructional and economical performances need to be satisfied simultaneously, which may lead to a completely different way of selecting the appropriate area ratio ρ from the ways govern by the mechanical parameters. The appropriate ranges of the ρ value for all the investigated seven mechanical parameters using the four analytical models of stay cable are summarized in Table 3.

From Table 3, it can be seen that it is difficult to select one ρ value to achieve an optimal performance of the stay cable with respect to all the parameters; however, an appropriate range can still be selected to achieve a good balance among the mechanical, constructional, and economical performances of stay cables. In the present study, by comparing the ρ values summarized in Table 3 a range of 0.55 to 0.85 is considered as an appropriate ρ range, taking all concerned performances of stay cables into account. However, the range of 0.55 to 0.85 is still somewhat wide and not convenient for designers to choose from for bridge engineering practices. In order to narrow down the appropriate range in different conditions of stay cables, factors that can affect the selection of ρ are studied in the following sections.

5. Other Factors Affecting the Design of ρ Value

The effects of the horizontally projected length of stay cables, cable force, and pylon height (i.e., vertically projected length of stay cables) on the selection strategy of the ρ value were taken into account in further parametric studies. These three parameters have been widely used as the main design parameters when designing traditional steel stay cables. Based on the suggested ρ range of 0.55 to 0.85, the effect of the three design parameters on the mechanical parameters of stay cables under three different ρ values (0.55, 0.70, and 0.85) were investigated. For these three stay cable models corresponding to the ρ values of 0.55, 0.70, and 0.85, respectively, the geometric and physical properties remain identical with those listed in Table 1 except the variables set for the parametric studies. It should be noted that a few cable cases (especially in the geometric factor) in this parametric study may not be suitable for current bridge design; however, the results from these studied cable cases are still important which can reflect the evolving trend of the area ratio ρ selection when varying the values of design parameters of composite stay cables. This trend can give the designer a quick guide for optimizing the area ratio when designing different composite stay cables.

5.1 Effects of the Horizontally Projected Length of Stay Cables

Varying the horizontally projected length of stay cables from 100 m to 3,000 m at 50 m intervals, the three analytical models for the proposed stay cable with ρ = 0.55, 0.70, and 0.85, respectively, were studied. By comparing the results from these models, the effects of the horizontally projected length of stay cables on the suggested ρ range of 0.55 to 0.85 obtained earlier can be investigated in detail. Different pylon heights were examined; however, almost the same effects as the horizontally projected length of stay cables can be obtained (see section 5.3). Due to the page limit, only the results from pylon height of 560 m are shown here.

Figures 8(a~g) show the performance of the stay cables in terms of the investigated mechanical parameters using the three ρ values. It is seen that in Fig. 8(a) the three curves are close to each other in the entire range of the horizontally projected length; in this situation, a low ρ value is preferred since it is more cost-effective but still can obtain approximately the same mechanical behaviors as that with a high area ratio ρ. However, in the remaining figures, it is shown that different appropriate ρ values should be individually chosen for different ranges of the horizontally projected length of stay cables. For example, in Fig. 8(f) for the equivalent elasticity modulus of stay cables, a lower ρ value of 0.55 can be set when the horizontally projected length is less than 1,500 m; however, with the increase of the horizontally projected length of stay cables the curve with the ρ value of 0.55 drops sharply, thus more CFRP materials in the composite section are required to keep a high equivalent elasticity modulus. For this purpose, a ρ value of 0.70 is used for the horizontally projected length of stay cables between 1,500 m to 2,500 m, and 0.85 is appropriate for the case exceeding 2,500 m.

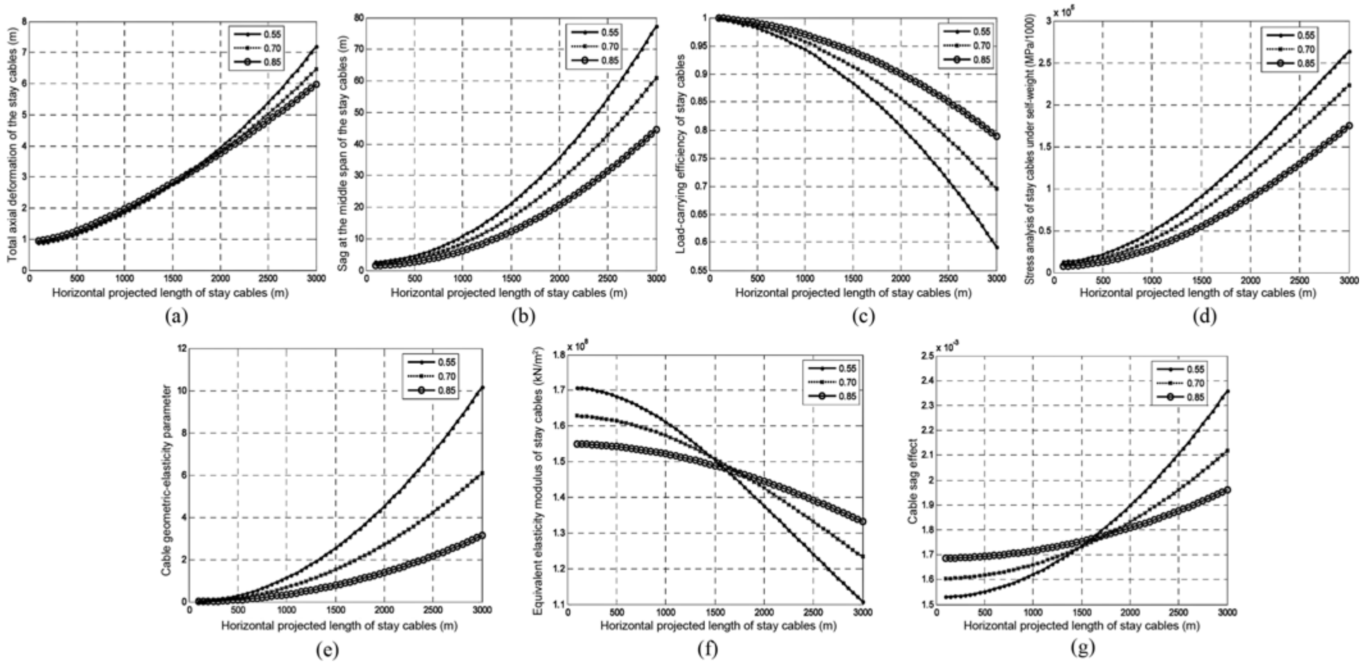

Fig. 8. Results of Parametric Study: (a) Total Axial Deformation of Stay Cables, (b) Sag at the Middle Span of Stay Cables, (c) Load-carrying Efficiency of Stay Cables, (d) Stress analysis of Stay Cables Under Self-weight, (e) Cable Geometric-elasticity Parameter λ^2, (f) Equivalent Elasticity Modulus of Stay Cables, (g) Cable Sag Effect

Following this approach, the appropriate horizontally projected lengths of stay cables for using the ρ value of 0.55, 0.70, and 0.85, respectively, can be individually obtained based on each parameter (see Table 4).

Taking all parameters into account, a conclusion can be drawn from Table 4 that a low ρ value (0.55) is appropriate when the horizontally projected length of stay cables is less than 1,000 m; however, a high ρ values, 0.70 and 0.85, is better for the cases where the horizontally projected lengths of stay cables are within the range of 1,000 m to 2,000 m, and exceeding 2,000 m, respectively.

5.2 Effects of Cable Force

Large cable forces provide extra stiffness to stay cables, which can significantly affect the mechanical behaviors of stay cables. In this section, the effects of cable force are considered when selecting an appropriate ρ value. For this purpose, a group of cable forces from 3,000 kN to 13,000 kN at 200 kN intervals were applied to the three analytical stay cable models with different ρ values (0.55, 0.70, and 0.85), horizontally projected lengths (500 m, 1,500 m, and 2,500 m), and vertically projected lengths (250 m, 750 m, and 1,250 m), respectively. The remaining material properties of the models are identical to those used in the previous parametric study. When selecting the ρ value for each analytical model with different horizontally projected lengths of stay cables, the conclusion of the previous study, i.e., the effects of the horizontally projected length of stay cables, was considered. By comparing the results from the three analytical cable models, the effects of cable forces on the suggested ρ range of 0.55 to 0.85 obtained earlier can be investigated in detail.

From the results of the seven parameters shown in Fig. 9, the results can be grouped into two categories for discussion. On one hand, as can be seen from Figs. 9(a), 9(b), and 9(e~g), there is a

Table 4. Appropriate Horizontally Projected Lengths of Stay Cables for Using Each ρ Value

Mechanical Behavior	Criterion	Appropriate horizontally projected lengths		
		ρ=0.55	ρ=0.70	ρ=0.85
Total axial deformation of stay cables	The lower is preferred	Whole range	None	None
Sag at the mid-span of stay cables	The lower is preferred	<1000 m	1000~2000 m	>2000 m
Load-carrying efficiency of stay cables	The higher is preferred	<1000 m	1000~1500 m	>1500 m
Stress analysis of stay cables under self-weight	The lower is preferred	<1000 m	1000~2000 m	>2000 m
Cable geometric-elasticity parameter λ^2	The lower is preferred	<1000 m	1000~2000 m	>2000 m
Equivalent elasticity modulus of stay cables	The higher is preferred	<1500 m	1500~2000 m	>2000 m
Cable sag effect	The lower is preferred	<1500 m	1500~2000 m	>2000 m

Note: a low area ratio ρ always obtains a good constructional and economical performance.

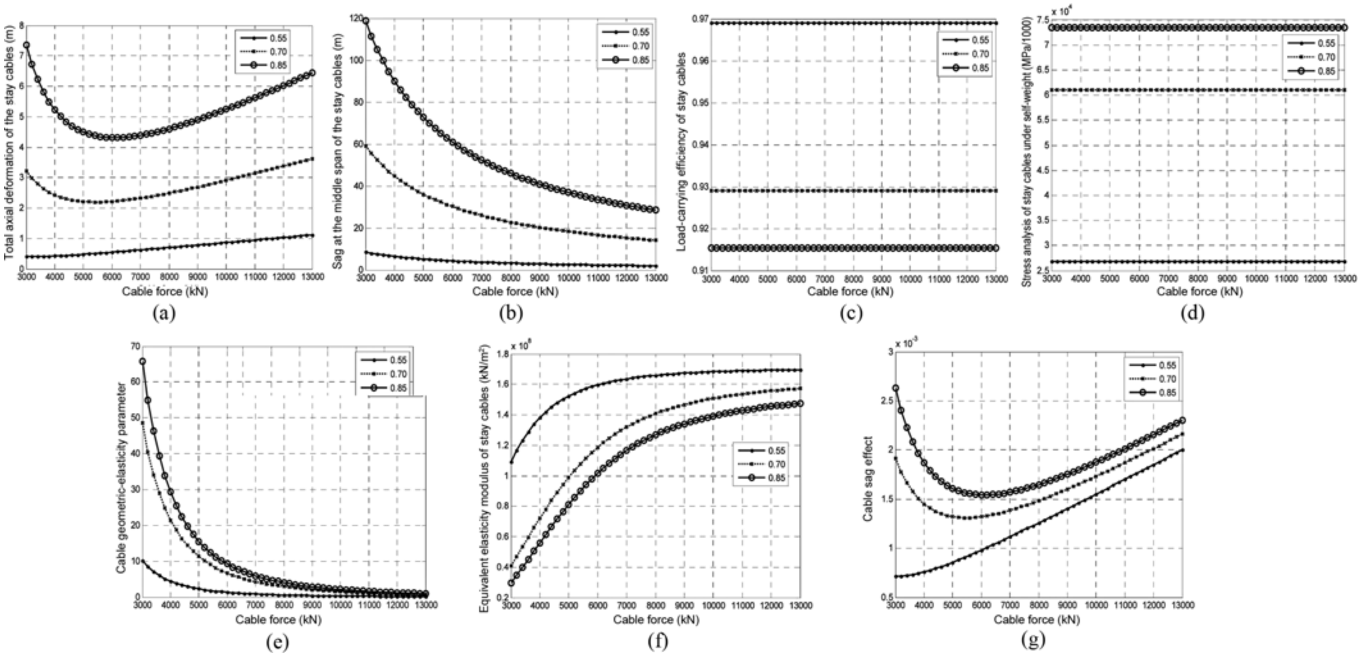

Fig. 9. Results of Parametric Study: (a) Total Axial Deformation of Stay Cables, (b) Sag at the Middle Span of Stay Cables, (c) Load-carrying Efficiency of Stay Cables, (d) Stress Analysis of the Stay Cable Under Self-weight, (e) Cable Geometric-elasticity Parameter λ^2, (f) Equivalent Elasticity Modulus of Stay Cables, (g) CablE Sag Effect

dramatic change in the investigated mechanical parameters as the cable force increases in the range of 3,000 kN to 6,500 kN. In the same range, for the identical cable force, significant difference exists in each investigated mechanical parameter calculated using the three cable models with different ρ values. Both phenomena indicate that the cable force has a significant effect on the mechanical behaviors of the stay cables when the cable forces are in that range. This is generally because that the mechanical behaviors in these figures are closely related to the axial stiffness of stay cables, and the axial stiffness will be sharply reduced by the cable sag effect due to the decrease of the cable force. As a result, for these cases, a high ρ value should be used to reduce the self-weight of stay cables and cable sag effect in order to increase the axial stiffness of stay cables.

However, as the cable force is greater than 6,500 kN, the curves in each figure gradually approach each other; as the cable force further increases, most of them approximately level off. In other words, within the range of over 6,500 kN the three analytical cable models with different ρ values perform similarly with respect to the cable-force-related mechanical behaviors. Therefore, a more cost-effective low area ratio ρ can still ensure a similar mechanical performance compared with that with a high area ratio ρ. In addition, the relationship between the cable sag effect and cable force can be described using Figs. 9(a), 9(b), and 9(g). When the cable force is below 6,500 kN the total axial deformation of stay cables is mainly due to the cable sag which is larger as shown by Fig. 9(b); however, while the cable force exceeds 6,500 kN the elastic extension of the stay cable itself will dominate the total axial deformation, as can be seen from Figs. 9(a) and 9(g).

On the other hand, from Figs. 9(c) and 9(d) it can be seen that changing the cable force does not have any effect on these two investigated mechanical behaviors. This can be due to the fact that the equations used for describing the two specific mechanical behaviors, i.e., load-carrying efficiency of stay cables and stress analysis of stay cables under self-weight in this paper, are not related to the cable force.

Based on the discussion above, it can be concluded that when the cable force is below 6,500 kN, a high ρ value (0.85) can be used to reduce the self-weight of stay cables and cable sag effect; however, when the cable force exceeds 6,500 kN, a lower area ratio ρ (0.55) can be selected to reduce the cost related to the material and manufacture for the proposed stay cables and also ensure a good mechanical performance.

5.3 Effects of Pylon Height

In the previous study of the effects of the horizontally projected length of stay cables, the possible effects of pylon height (i.e., vertically projected length of stay cables) on the selection of ρ values were not introduced and the value of 560 m was used as a constant pylon height. In this section, the effects of the pylon height were investigated using other two analytical cable models with two different pylon heights of 287 m and 1,000 m. Figs. 10(a)~(g) and Figs. 11(a)~(g) show the results for the 287 m and 1000 m pylon heights, respectively.

From Figs. 10(a)~(g) and 11(a)~(g), it can be clearly seen that the results for the 287 m and 1,000 m pylon heights are very similar to those of the 560 m pylon height obtained previously. In other words, the pylon height has little effect on the selection of the appropriate ρ values. One can see that the conclusion obtained in the previous study regarding the effects of the horizontally

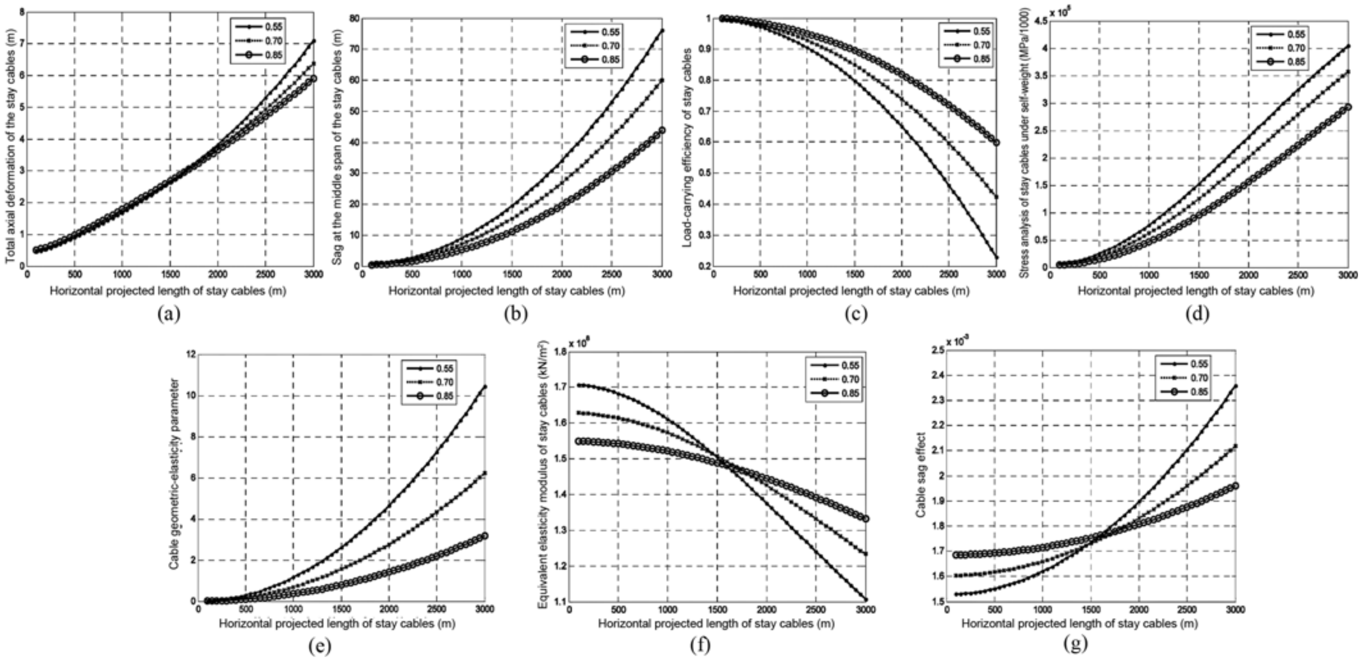

Fig. 10. Results of Parametric Study Under 287 m Pylon Height: (a) Total Axial Deformation of Stay Cables, (b) Sag at the Middle Span of Stay Cables, (c) Load-carrying Efficiency of Stay Cables, (d) Stress Analysis of stay Cables Under Self-weight, (e) Cable Geometric-elasticity Parameter λ^2, (f) Equivalent Elasticity Modulus of Stay Cables, (g) Cable Sag Effect

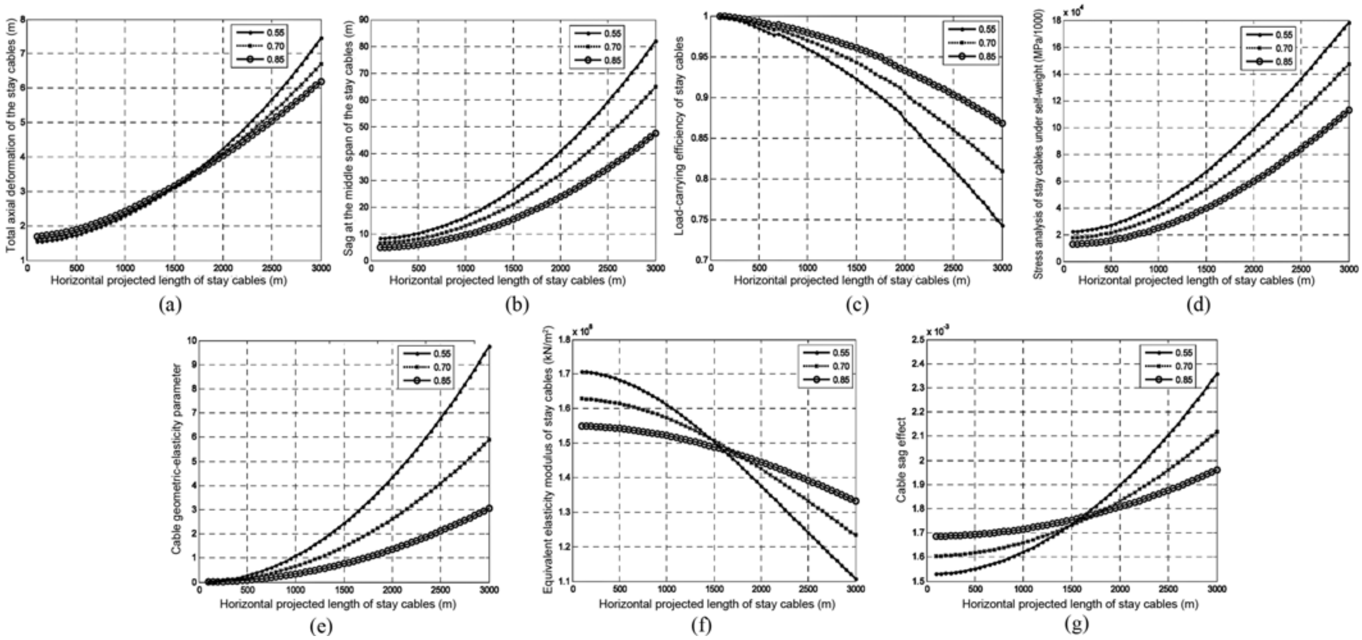

Fig. 11. Results of Parametric Study Under 1,000 m Pylon Height: (a) Total Axial Deformation of Stay Cables, (b) Sag at the Middle Span of Stay Cables, (c) Load-carrying Efficiency of Stay Cables, (d) Stress Analysis of Stay Cables Under Self-weight, (e) Cable Geometric-elasticity Parameter λ^2, (f) Equivalent Elasticity Modulus of Stay Cables, (g) Cable Sag Effect

projected length of stay cables is still valid for the stay cables of cable-stayed bridges with other reasonable pylon heights.

6. Feasibility of the Proposed Stay Cables

In this section, the feasibility of the proposed stay cables was conceptually verified in two ways: the possibility of fully utilizing the total strengths of both CFRP and steel materials under the ultimate loads and the redistribution of the internal force between the two materials under the temperature changes. For the first one, because the ultimate strength of CFRP materials is always achieved after the yielding but before the break of the steel materials (AASHTO 2004; ACI 440.4R-04 2004), CFRP core can still carry external loads after the steel coat has yielded until it fails under the ultimate loads. Therefore, both CFRP and steel materials can fully utilize their total strengths. The latter one was studied through calculating the redistributed internal force in the proposed stay cables and comparing it to the tensioned cable forces.

For the materials used in the proposed stay cables in this paper, from Table 1, the axial thermal coefficient of expansion of CFRP materials (0.68E-6/ºC), is only one seventieth of that of steel materials (11.7E-6/ºC); for the transverse thermal coefficient of expansion, CFRP materials (11.7E-6/ºC) is half of the steel materials (25.0E-6/ºC). Due to the obvious difference in the thermal coefficient of expansion between CFRP and steel materials, it is worthy of studying the internal force redistribution (or the corresponding strains) under the temperature-induced loads for the proposed stay cables and the process for calculating the redistributed force is demonstrated next.

It is in general safe and conservative to assume that CFRP core is not shortened or stretched under small temperature change due to the relatively low thermal coefficient of expansion compared to the steel coat. As a result, to achieve displacement compatibility between the two materials an interacting force at the interface between the two constituents can be approximately calculated as:

$$F = \alpha \cdot \Delta \cdot E \cdot A_{STEEL} \quad (8)$$

where F = redistributed internal force; α = axial thermal coefficient of expansion of steel materials; Δ = temperature variation; E = elasticity modulus of steel materials; and A_{STEEL} = steel area.

All values could be determined from Table 1 except A_{STEEL} and Δ which are assumed to be 40% of the total area of cross section and 20ºC, respectively. The redistributed force can then be calculated as:

$$F = \alpha \cdot \Delta \cdot E \cdot A_{STEEL} = (11.7\text{E-}6)\times 20\times(2.00\times 10\text{E}8)\times(0.015\times 0.4)=280.8\text{ kN} \quad (9)$$

It should be noted that the calculated value (280.8 kN) could be slightly greater than the actual redistributed internal force due to the assumptions made.

Compared with the results from the preliminary designs of the 1400 m span cable stayed bridge by other researchers (Nagai *et al*., 2004; Lin, 2004; Miao, 2006), where most of the cable forces were designed as larger than 4,000 kN and even up to 8,000 kN sometimes, the calculated redistributed internal force is far less than the tensioned cable forces. In other words, it can be concluded that the temperature-induced internal force redistribution cannot cause significant effects on the practical applications of the proposed stay cables. The thermal strains in CFRP and steel materials due to the temperature-induced internal force redistribution should be very small and the corresponding slipping between the two constituents can be simply ignored. It should be noted that although the discussion is only given for CFRP and steel constituents, the analysis should be totally the same when the bonding is taken into consideration.

Based on the analysis above, the feasibility of the proposed stay cables can be simply verified.

7. Conclusions

In this paper, stay cables with a CFRP and steel composite section were proposed and theoretically studied through parametric studies. The study can be summarized as:

In the concept of the proposed stay cables, the CFRP core conserves all the advantages of CFRP materials such as light weight and high strength; while the steel coat provides a protection for the CFRP core for a reliable anchorage performance, increases the elastic modulus, and reduces the total cost.

The area ratio ρ is considered as a key design parameter for the composite section of the cable stays. Based on the results from the parametric study, 0.55~0.85 can be set as an appropriate ρ range by taking into account all the mechanical, constructional, and economical performances.

Considering the effects of the horizontally projected length of stay cables, a low ρ value (0.55) is appropriate when the horizontally projected length of stay cables is less than 1,000 m; 0.70 is better for the horizontally projected lengths of stay cables within 1,000~2,000 m; and 0.85 is for the horizontally projected lengths of stay cables over 2,000 m.

When the cable force is below 6,500 kN, a high ρ value (for example, 0.85) can be used to reduce the self-weight of stay cables and cable sag effect. However, when the cable force exceeds 6,500 kN, a low area ratio ρ (for example, 0.55) can be selected to reduce the cost for the proposed stay cables and also ensure a good mechanical performance.

The effect of the pylon height (vertically projected length of stay cables) can be simply ignored when selecting the appropriate ρ value since it does not affect the performance of the analytical cable models.

The final area ratio ρ of the composite section should be always adjusted with the consideration of the construction factor such as anchorage.

The proposed stay cable is feasible because of its ability to fully utilize the total strengths of CFRP and steel materials and the low redistributed internal force between the two materials under the temperature-induced loads.

Based on the theoretical study, this novel composite stay cable can be an excellent alternative to the pure CFRP stay cables or traditional steel stay cables, especially for the large bridges crossing oceans. It should be noted that the present study on the composite stay cables is still conceptual. Future work includes the studies of specific bonding mediums/techniques between CFRP and steel materials, slip analysis on the interface between CFRP and steel materials, transportation method of CFRP materials to the situ, etc. A comparative study of the proposed stay cables to the pure CFRP or traditional steel stay cables is also needed using a finite element model of a cable stayed bridge structure.

Acknowledgements

The first author appreciates the financial support from the Chinese Government. This scholarship makes his research at Louisiana State University possible. The financial support from the National Natural Science Foundation of China (Project No. 50828803) is also gratefully acknowledged. The opinions and statements do not necessarily represent those of the sponsors.

References

AASHTO LRFD Bridge Design Specifications (2004). *Association of state highway and transportation officials*, Washington, D.C., U.S.A.

Ahmadi-Kashani, K. and Bell, A. J. (1988). "The analysis of cables subject to uniformly distributed loads." *Engineering Structures,* Vol. 10, No. 3, pp. 174-184.

American Concrete Institute (ACI) (2004). *Prestressing concrete structures with FRP tendons (ACI 440.4R-04)*, Detroit, U.S.A.

Burtscher, S. L. (2008). "Wedge anchorage for CFRP strips." *Journal of Composites for Construction,* Vol. 12, No. 4, pp. 446-453.

Cai, C. S., Wu, W. J., and Shi, X. M. (2006). "Cable vibration reduction with a hung-on TMD system. Part I: Theoretical study." *Journal of Vibration and Control*, Vol. 12, No. 7, pp. 801-814.

Colombi, P. and Poggi, C. (2006). "Strengthening of tensile steel members and bolted joints using adhesively bonded CFRP plates." *Construction and Building Materials,* Vol. 20, Nos. 1-2, pp. 22-33.

Ernst, H. J. (1965). "Der e-modul von seilen unter beruecksichtigung des durchhanges." *Der Bauingenieur,* Vol. 40, No. 2, pp. 52-55.

Freire, A. M. S., Negrao, J. H. O., and Lopes, A. V. (2006). "Geometrical nonlinearities on the static analysis of highly flexible steel cable-stayed bridges." *Computers and Structures,* Vol. 84, Nos. 31-32, pp. 2128-2140.

Gimsing, N. J. (1997). *Cable-supported bridges (2nd Edition)*, John Wiley & Sons, New York, U.S.A.

Irvine, H. M. (1981). *Cable structures*, The MIT Press, Cambridge, MA, U.S.A.

Jayaraman, H. B. and Knudson, W. C. (1981). "A curved element for the analysis of cable structures" *Computers & Structures,* Vol. 14, Nos. 3-4, pp. 325-333.

Lin, Y. P. (2004). *Cable-stayed bridge*, China Communications Press, Beijing, China.

Matta, F. (2003). *Bond between steel and CFRP laminates for rehabilitation of metallic bridges*, PhD Thesis, Faculty of Engineering, University of Padua, Padua, Italy.

Mei, K. H. (2005). *Study on cable-stayed bridges with CFRP cables*, PhD Thesis, Dongnan University, Nanjin, Jiangsu. China.

Meier, U. (1987). "Proposal for a carbon fiber reinforced composite bridge across the strait of Gibraltar at its narrowest site." *Proc. Inst. Mech. Eng.,* Vol. 201, No. B2, pp. 73-78.

Miao, J. W. (2006). "Study for design theories of super-long span cable-stayed bridges." PhD Thesis, Tongji University, Shanghai, China.

Nagai, M., Fujino, Y., Yamaguchi, H., and Iwasaki, E. (2004). "Feasibility of a 1,400 m span steel cable-stayed bridge." *Journal of Bridge Engineering,* Vol. 9, No. 5, pp. 444-452.

O'Brien, T. W. and Francis, A. J. (1964). "Cable movements under two-dimensional loading." *ASCE J. Struct. Engng.,* Vol. 90, No. ST3, pp. 89-123.

Peyrot, A. H. and Goulois, A. M. (1978). "Analysis of flexible transmission lines." *ASCE J. Struct. Div.,* Vol. 104, No. 5, pp. 763-779.

Rizkalla, S., Dawood, M., and Schnerch, D. (2008). "Development of a carbon fiber reinforced polymer system for strengthening steel structures." *Composites Part A: Applied Science and Manufacturing,* Vol. 39, No. 2, pp. 388-397.

Saadatmanesh, H. and Ehsani, M. R. (1998). "Fiber composites in infrastructure." In *2nd International Conference ICCI,* 05-07 January 1998, Tucson, Arizona, U.S.A.

Sayed-Ahmed, E. Y. (2002). "Single- and multi-strand steel anchorage systems for CFRP tendons/stays." *Proceedings, Annual Conference - Canadian Society for Civil Engineering,* pp. 1381-1390.

Schnerch, D., Stanford, K., Sumner, E. A., and Rizkalla, S. (2004). "Strengthening steel structures and bridges with high modulus carbon fiber reinforced polymers: Resin selection and scaled monopole behavior." *TRB 2004 Annual Meeting- Transportation Research Record,* Washington, DC, USA.

Schurter, U. and Meier, B. (1996). "Stork bridge winthertur." *Schweizer Ingenieur und Architekt 44*, October 1996, 976-979.

Sen, R., Liby, L., and Mullins, G. (2001). "Strengthening steel bridge sections using CFRP laminates." *Composites Part B: Engineering,* Vol. 32, No. 4, pp. 309-322.

Zhao, X. L. (2008). "FRP strengthening of thin-walled structures." *CIMS2008, Fifth International Conference on Coupled Instabilities in Metal Structures, Gregory J Hancock Symposium,* June, 2008, Sydney, Australia, pp. 23-25.

Zhao, X. L. and Zhang, L. (2007). "State-of-the-art review on FRP strengthened steel structures." *Engineering Structures,* Vol. 29, No. 8, pp. 1808-1823.