# Assessment of Structural Robustness under Different Events according to Vulnerability

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**Abstract:** Structural robustness is investigated in this study according to its opposite property, i.e., structural vulnerability, which is calculated as the assembly of component vulnerability measured by the vulnerability coefficient. Different types of vulnerability coefficients are suitable for both truss and RC frame structures. An importance coefficient based on the bearing capacity of the remaining structure is proposed to reveal the internal topology and failure scenarios, and is viewed as the weight coefficient of the corresponding vulnerability coefficient. The occurrence probability density function is introduced to describe the uncertainty of abnormal events and assess structural robustness under different events. Numerical examples of several idealized trusses and a RC frame are performed to demonstrate the use of the proposed robustness index. Analysis results show that the robustness index provided good explanation for robustness quantification for both truss and RC frame structures under different events. Moreover, two methods, i.e., increasing the local resistance and redundancy of the structure, to upgrade structural robustness also testify to the effectiveness and accuracy of the robustness index. **DOI: 10.1061/(ASCE)CF.1943-5509.0000854.** 

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## Introduction

The definition of structural robustness has been controversial since its first proposal after the collapse of the Ronan Point apartment in 1968. Biondini et al. (2008) viewed structural robustness as the ability of a system to endure an amount of local damage not disproportionate to the causes of the damage. The U.K. Standing Committee on Structural Safety (SCOSS 1994) defined structural robustness as the capability to resist disproportionate collapse. Robustness has also been defined as the insensitivity of a structure to initial damage. Ellingwood (2006) considered robustness a fundamental property of structural systems to prevent the occurrence of damage propagation phenomena and to mitigate risks from disproportionate failure events and progressive collapse. The consensus in recent years is that a structure is considered robust when an initial damage of part of the structure does not lead to disproportionate collapse of the entire structure (Starossek et al. 2011).

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Early research on structural robustness was qualitative. The U.K. Standing Committee on Structural Safety emphasized, in its 10th report, the need to protect structures against progressive collapse. McGuire (1974) stated that it was not suitable to consider the alternative load paths or to enhance local resistance as the best choice to prevent structural collapse, and it should be achieved by structural integrity requirements. Ellingwood and Leyendecker (1978) summarized three main ways against progressive collapse: accident control, indirect design, and direct design. These studies were all qualitative.

Recent research has focused on quantifying structural robustness. The quantification indexes of robustness are divided into three categories, namely, deterministic performance-based, reliability-based, and risk-based indexes, which all reveal structural robustness to some extent. The deterministic performance of a structure involves several aspects such as load-carrying capacity and stiffness, based on which many corresponding robustness measures have been proposed. A residual influence factor was proposed to measure the effect of the failure of structure member $i$ on the load-carrying capacity of an intact structure (Słrensen 2011). Similar robust measures include the displacement-based robustness index proposed by Biondini et al. (2008) and the stiffness-based robustness index proposed by Starossek and Haberland (2009). These robustness indexes describe the attribute change of a structure with and without the removal of elements. Frangopol and Curley (1987) and Fu and Frangopol (1990) considered system reliability to quantify structural redundancy (a system property that largely accounts for structure robustness), according to the relationship between damage probability and system failure probability. Baker et al. (2008) built a risk-based framework for robustness assessment, within which consequences were classified as either direct or indirect. Robustness index was defined as the ratio of direct consequences to the sum of direct and indirect consequences. This work provided a systematic method for the quantification of robustness, but it was difficult to be generalized to other complicated types of system in that the distribution of applied load and individual component resistances depended on the subjective assumption of researchers.

Though there have been a large number of researches on structural robustness, there are still no well-established and generally accepted criteria for a consistent definition and a quantitative measure of structural robustness.

In fact, the definition of structural robustness actually contains two aspects to be solved: (1) uncertainty (Kim and Taha 2009) of abnormal events, and (2) ability of the structure to resist collapse when locally damaged, which is a system property relevant to the form and connectivity of a structure. Engineering structures are exposed to a complex environment in which terrorist attacks and natural hazards such as earthquakes, landslides, and hurricanes could occur at different probabilities based on the location, configuration, and function of structures. The uncertainty of abnormal events has elicited much research attention and has resulted in the quantification of robustness being considered from the perspective of the probabilistic method (Ziha 2001). However, most indexes are not applicable enough for practical engineering because it is difficult to consider all foreseeable events within a single robustness index. Moreover, the process of progressive collapse (Helmy et al. 2013) is also usually neglected and the internal topological relations cannot be revealed by those indexes that consider only uncertainty. Thus, it is more reasonable to first develop a robustness index to reflect how the structure responses to a specific event (Qian and Li 2012, 2013; Kaewkulchai and Williamson 2006) and further generalize it to multiple hazard scenarios.

In this study, the uncertainties of a structure, including the variation of materials and geometry dimensions, are not considered. The purpose of this study is to propose a calculable, expressive, and general (Starossek and Haberland 2008, 2009) robustness index that accounts for both the topology of the structure and assessment of robustness under specific corresponding events. The robustness index is obtained by quantifying the opposite property of robustness, i.e., structural vulnerability, through component importance and vulnerability coefficients.

## Division of Progressive Collapse

Progressive collapse can be divided into four stages, as shown in Fig. 1:

1. Undamaged stage (US): The intact structure was exposed to normal circumstances, and all components were able to resist the design load. Robustness assessment could not be performed because the robustness referred to in the present study was relevant to specific events, and no abnormal events happened within zone US.
2. Locally damaged stage (LDS): The moment when the structure suffered an unexpected explosion, an exterior column failed in no time due to the tremendous impact wave, followed by an instantaneous response from the entire structure in a short period [zone LDS in Fig. 1(b)], within which internal forces were formed in all components, and insufficient time was left for deformations to develop. This slightly and locally damaged stage is selected to evaluate the vulnerability of the structure in most cases because it is closest to the original intact structure.
3. Damage propagation stage (DPS): It took a relatively long time (compared with zone LDS) for the explosives load to partially or totally damage the structure. During the damage propagation stage, subsequent damage propagated elsewhere in the structure, and an increasing number of components failed due to the redistribution of internal forces. Total collapse would have likely occurred had the structure been insufficiently redundant. The presence of segment borders (i.e., compartmentalization) would

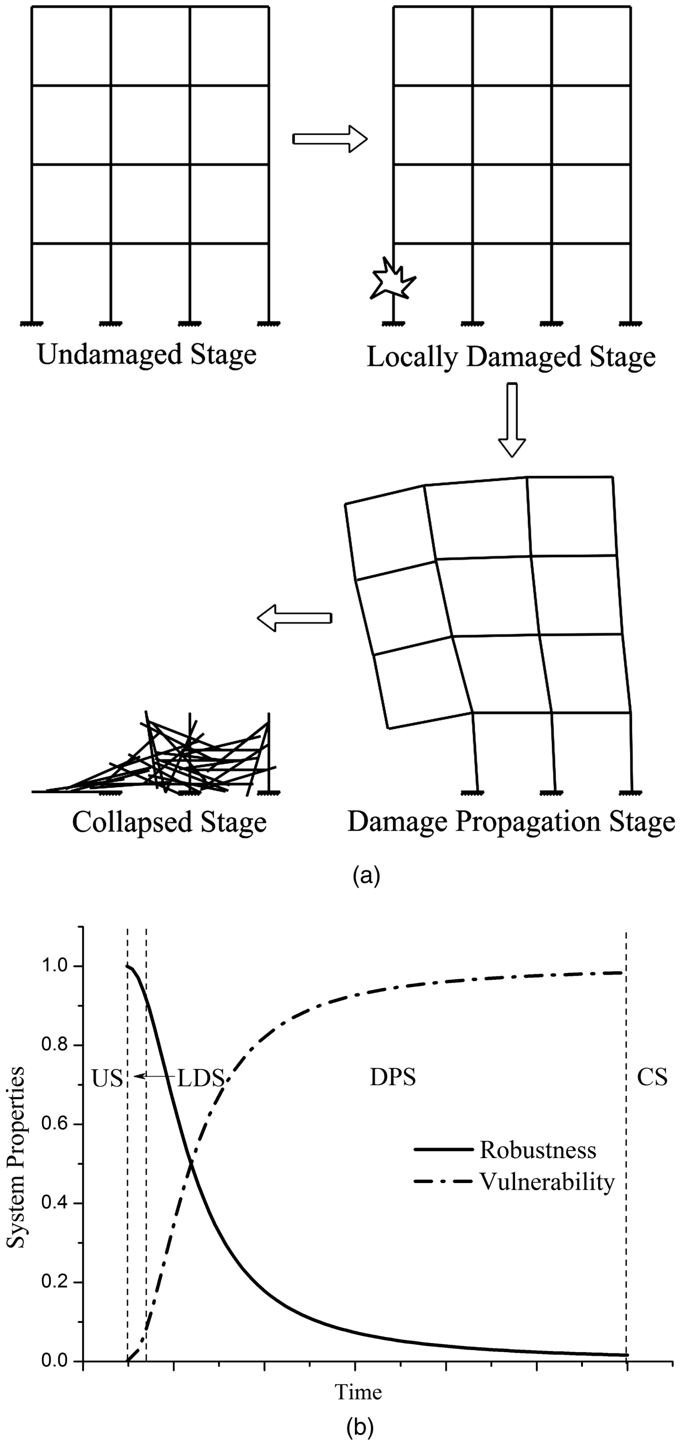


**Fig. 1.** Division of progressive collapse: (a) the four stages; (b) change in system properties

have led to partial collapse in that damage spread stopped at the segment border and was confined to a relatively small region. Every failure of a component means that the form and connectivity of the structure have changed; thus, the system properties of a newly formed structure change with time. Generally, structural robustness deteriorates, whereas the vulnerability of the structure, which is the opposite of robustness, increases until the end of the collapse.

4. Collapsed stage (CS): The structure responded no more and the collapse of the structure ended. Rescue measures should be

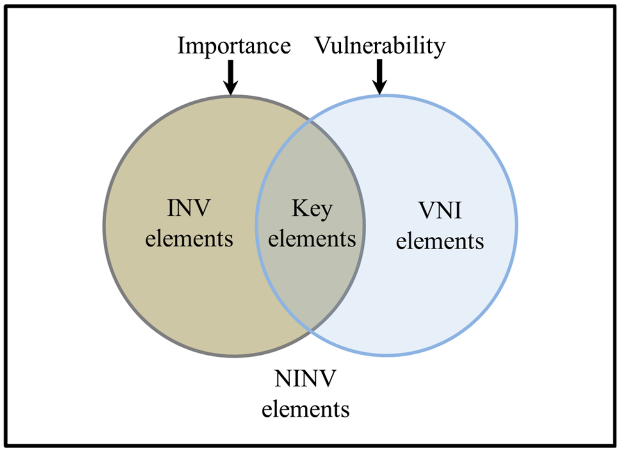


**Fig. 2.** Venn diagram of the classification of components under a certain event

immediately implemented in case of postevent hazards, such as aftershock.

## Roles of a Component

A structure is regarded as an assembly of components in this study. The progressive collapse of an entire structure is the macro manifestation of behaviors of components. In this context, it is reasonable that the system property of structure is investigated from the component perspective. A component within a structure plays two roles at the same time. As a single component, it should possess sufficient local resistance to be invulnerable when under abnormal events. As a part of a structure, the failure of a key component may lead to subsequent failure of other components or even the collapse of the entire structure. This condition indicates that components are of different importance to the structure. Components are classified into four groups according to their degree of vulnerability and importance, as shown in Fig. 2:

- Key elements refer to elements with high importance that are vulnerable to a certain event. For instance, an explosion at a bottom exterior column may lead to disproportional collapse of the affected structure, had the structure failed to bridge over the effects induced by the loss of a single column.
- Important but not vulnerable (INV) elements refer to elements with high importance but not vulnerable to a certain event. Strengthening of a bottom interior column during the design stage is important to transfer the upper load to the foundation, and can improve the capacity to resist unexpected loads.
- Vulnerable but not important (VNI) elements are of minimal importance and vulnerable to a certain event. Secondary nonstructural elements belong to this group.
- Neither important nor vulnerable (NINV) elements are neither important nor vulnerable to a certain event. From the perspective of mechanics, NINV elements are uneconomical mainly because of design mistakes. For instance, a secondary element like the upmost beam might possess a stronger section than that of the bottom column due to a lack of experience of the designer. NINV elements are able to resist the external load, but it is unreasonable and thus should be avoided.

Distinguishing components is essential to determine structural vulnerability under the different contributions provided by individual components. Importance and vulnerability coefficients are introduced to illustrate how this goal is achieved.

## Vulnerability Coefficient

A vulnerability coefficient is a measure of component vulnerability. Whether a component is vulnerable or not mainly depends on the failure criteria of the component. The larger internal forces resulting from the external load, the more likely it is the component will fail. The failure criteria of a component varies depending on the structural type (truss or RC frame).

### *Vulnerability Coefficient of the Truss Component*

Truss components carry only axial force; thus, the failure criteria can be determined from the stress or strain level. Components made of two types of material, i.e., brittle material and elastic-perfectly plastic material, were considered.

A truss component made of a brittle material does not fail unless the maximal elastic strain is reached. Therefore, the vulnerability coefficient is defined as follows:

$$\nu = \varepsilon/\varepsilon_{emax}, \qquad 0 \le \varepsilon \le \varepsilon_{emax} \tag{1}$$

where $\varepsilon$ and $\varepsilon_{emax}$ = axial strain resulting from external load and maximal elastic strain of the brittle material, respectively.

The vulnerability coefficient can be rewritten as Eq. (2), which is the internal force ratio utilized to judge whether a truss component failed or not in the previous part of the importance coefficient. Both vulnerability coefficients in the form of strain and axial force are equivalent

$$\nu = F/N_{\mathrm{max}} \tag{2}$$

Similarly, for a truss component made of an elastic-perfectly plastic material, the corresponding vulnerability coefficient is defined as follows:

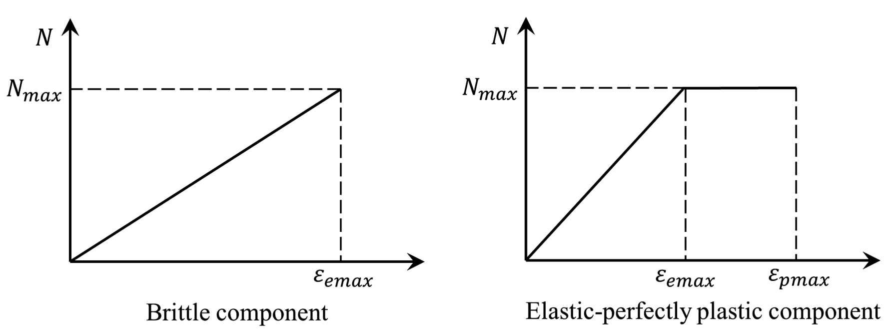


**Fig. 3.** Axial force and strain relation of two kinds of components

$$\nu = \varepsilon/\varepsilon_{p\max}, \qquad 0 \le \varepsilon \le \varepsilon_{p\max} \tag{3}$$

where $\varepsilon$ and $\varepsilon_{p\max}$ = axial strain resulting from external load and the maximal plastic strain of the elastic-perfectly plastic, respectively. However, Eq. (3) cannot be rewritten in the form of axial force ratio because the axial force is not always proportionate to the strain, as depicted in Fig. 3.

### Vulnerability Coefficient of the RC Component

RC members subjected to the combined loadings of bending, shear, and axial compression (tension) are very common in structures. The distribution of strain along the length of a RC member can be fairly complex, making it impossible to use the stress or strain as the failure criteria. Considering the interactions (Cesare and Archilla 2006) of load effects in beams and columns, an interaction formula (Huang et al. 2013) containing a strength envelope (Fig. 4) can be utilized as failure criteria of RC components to determine the vulnerability of members

$$\begin{gathered} p\left(\frac{0.5V}{V_0}\right)^2 + p\left(\frac{M}{M_0}\right) + q\left(\frac{N}{N_0} - h\right)^2 = 1 \\ \left(\frac{0.5V}{V_0}\right)^2 + \frac{M}{M_0} + \frac{P}{P_0} = 1 \end{gathered} \tag{4}$$

where $N_0$, $P_0$, $M_0$, and $V_0$ = ultimate capacity of the section under pure axial tension, axial compression, bending, and shear, respectively; and $N$, $P$, $M$, and $V$ = internal forces of the interactive section resulting from external load. The factors $p$, $q$, and $h$ (Lu et al. 2015) are calculated as Eq. (5)

$$\begin{gathered} p = -\frac{4k}{(k-1)^2}, \qquad q = \frac{4k^2}{(k-1)^2}, \qquad h = \frac{k+1}{2k} \\ k = \frac{n+m-1}{n^2-n} \qquad n = \frac{N_b}{N_0}, \qquad m = \frac{M_b}{M_0} \end{gathered} \tag{5}$$

where $N_b$ and $M_b$ = ultimate axial compression capacity and ultimate bending capacity in balanced conditions when subjected to eccentric compression, respectively.

Each point in Fig. 4 represents a state of internal forces of a RC component. Within the strength envelope, the closer the point is to the envelope surface, the more vulnerable the component will be. A point beneath the envelope surface (Point $P_1$) means that the component is in a safe state and can bear additional loads. A point on the envelope surface (Point $P_2$) means that the component is in a limit state. A point that exceeds the envelope surface (Point $P_3$) means the component has failed. Therefore, the vulnerability coefficient of a RC component is given as Eq. (6). Selecting between the two situations depends on whether a component is under the interaction of axial compression, bending, and shear or under the interaction of axial tension, bending, and shear

$$\nu = \begin{cases} p\left(\frac{0.5V}{V_0}\right)^2 + p\left(\frac{M}{M_0}\right) + q\left(\frac{N}{N_0} - h\right)^2, & N = \text{compression} \\ \left(\frac{0.5V}{V_0}\right)^2 + \frac{M}{M_0} + \frac{P}{P_0}, & P = \text{tension} \end{cases} \tag{6}$$

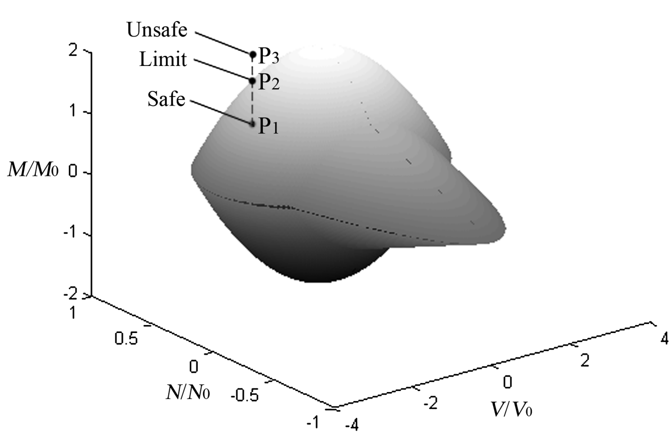


**Fig. 4.** Strength envelope of a symmetrically reinforced concrete component (data from Lu et al. 2015)

### Stage for Calculation of Vulnerability Coefficients

The value of a vulnerability coefficient is consistent with the external load. No component could be deemed as indestructible when the load is sufficiently large. Thus, the determination of the vulnerability of different components should involve the precondition that all components are under the effect of the same specified event. As introduced in "Division of Progressive Collapse," the form and connectivity of a structure change with the external load during the damage propagation stage; the locally damaged stage is closest to the original intact structure and is thus selected for assessment of structural vulnerability. In fact, local damage to a structure is often accompanied by the deterioration of the bearing capacity of that structure in most cases. Therefore, the appropriate stage for vulnerability evaluation is such a moment when the bearing capacity of a structure begins to deteriorate.

Specifically, for a truss structure composed of brittle members, failure of the first component often leads to a reduction in the bearing capacity of the structure in most cases and indicates the stage for vulnerability evaluation. However, there are still very few cases wherein the bearing capacity of a structure increases after the failure of a component. This rare phenomenon will be explained subsequently through a numerical example. When the stage of vulnerability evaluation is ascertained, the vulnerability coefficients of the structural components can be calculated according to the internal forces of components and the corresponding failure criteria.

## Importance Coefficient

As mentioned previously, components are of different importance to a structure. Dutuit and Rauzy (2001) proposed a critical importance factor that depends on component reliability, as a measure of component criticality. Nafday (2008) defined the importance measure for the removed member as the ratio of the volume of the normalized system stiffness matrix for the intact condition to the volume under the damaged condition. Failure scenarios and structural topology are not of concern in both of the two importance measures. A new type of component importance coefficient based on structural bearing capacity is proposed in this study. The proposed coefficient is represented as Eq. (7)

$$\gamma_i = \frac{R_0 - R_i}{R_0} = 1 - \frac{R_i}{R_0} \tag{7}$$

where $\gamma_i$ = importance factor of component $i$; $R_0$ = initial structural bearing capacity; and $R_i$ = structural bearing capacity after the failure of component $i$.

The preceding component importance coefficient can reflect the variation in structural bearing capacity before and after local failure, which is very similar to the meaning of structural robustness. The calculation of importance coefficients in the succeeding section shows how it reflects failure scenarios (England et al. 2008; Kanno 2012) and bridges are the topologic relation between local components and the overall structure.

### Truss Component

The importance coefficient of components depends on the distribution of external loading, which determines the bearing capacity of a structure. Therefore, when comment is made on the importance of a column or beam, it refers to its component importance coefficient under a certain event. As one of the simplest structural types, truss is an ideal model for illustration. Incremental load analysis was applied to a truss to determine its failure scenarios. This example is based on the following three assumptions:

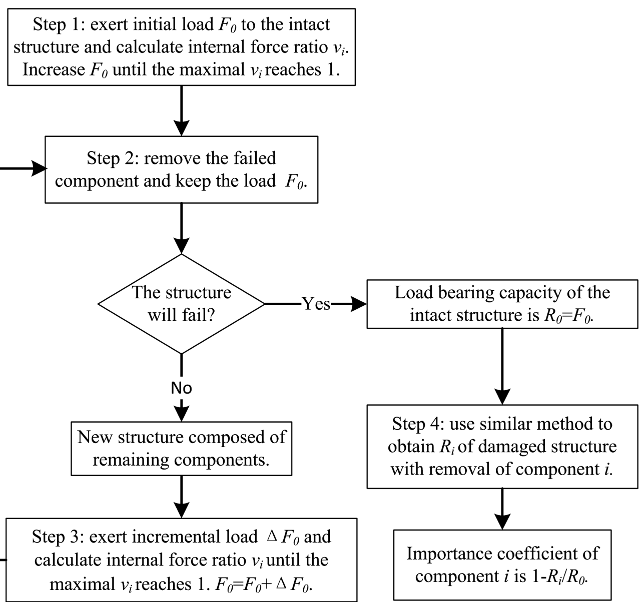


**Fig. 5.** Computation process of the importance coefficient of truss components

1. All truss members are brittle, and the ultimate axial compression and tension capacities are equal to each other. A component will fail and be removed if the axial force reaches its ultimate axial compression (tension) capacity.
2. Buckling does not occur in the components subjected to compression load.
3. The removal of a component is notional, i.e., no dynamic effects caused by the removal are considered. This is different from the actual situations, such as earthquake, explosion, and fire, where falling debris exerts an impact load on other undamaged components.

According to Eq. (7), the bearing capacities of both intact and damaged structures are the two main problems to be solved. Suppose that the structure is composed of $n$ components and $N_{\mathrm{imax}}$ denotes the ultimate axial compression (tension) capacity of component $i$. The process of calculating the importance coefficients of truss components is shown in Fig. 5 and is explained in the following steps:

1. A certain load distribution is applied to the intact structure, which defines a certain event. External load $F_0$ is increased and the internal force ratio $\nu$ of each component (i.e., the ratio of internal force $F$ to $N_{\max}$) is calculated. The corresponding $F_0$ is recorded when the maximal internal force ratio reaches the value of 1, which means a certain component failed;
2. The failed component is removed, and load $F_0$ is kept; if the structure becomes a mechanism immediately or after the subsequent failure of other components, then the bearing capacity of the intact structure $R_0$ equals $F_0$. Afterwards, proceed to Step 4. If the structure survives under the current $F_0$, it becomes a new structure composed of the remaining components; afterwards, proceed to Step 3;
3. An incremental load, $\Delta F_0$, is applied to the new structure. The internal force ratios are calculated. The application of incremental loads is stopped when the internal force ratio(s) of certain a component(s) reaches 1. The previous $F_0$ is updated with the addition of $\Delta F_0$, i.e., $F_0 = F + \Delta F_0$; then, return to Step 2; and
4. Component $i$ is removed, and a damaged structure is formed, to which the same event, i.e., the same load distribution, is applied. The bearing capacity of damaged structure $R_i$ can be obtained, through the same incremental method applied to the intact structure. So far, the importance coefficient of component $i$ has been obtained, i.e., Eq. (7). To obtain all other importance coefficients, Step 4 is repeated until the removal of all components is exhausted.

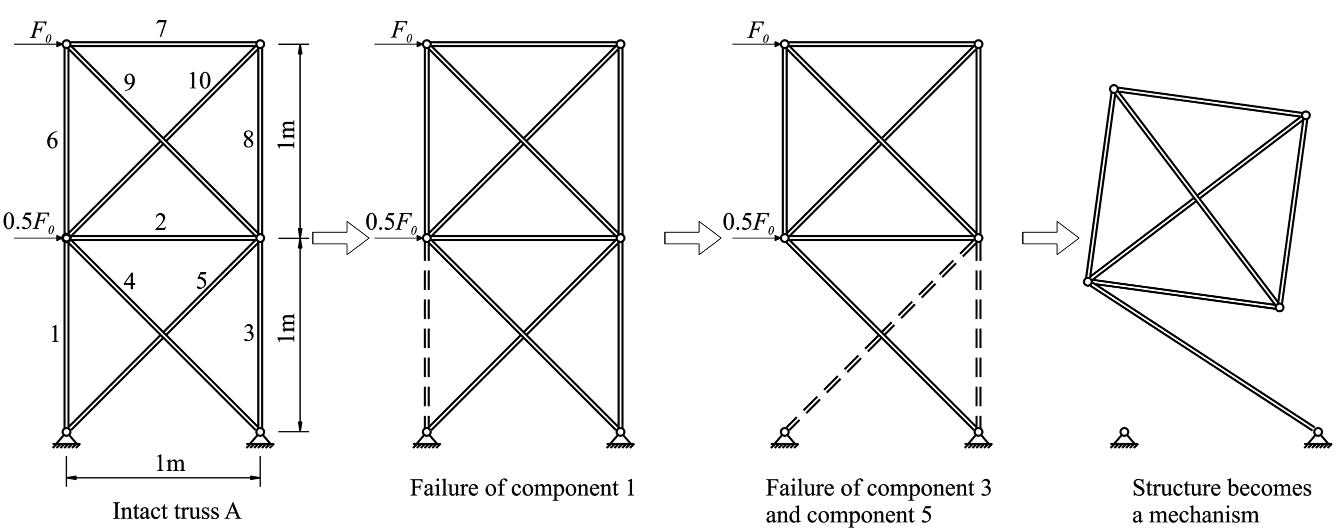


**Fig. 6.** Failure scenarios of Truss A

The truss structure is shown in Fig. 6 as an example, and the details of section properties are shown in Table 1. All components share the same section properties because the truss optimization design is insignificant in this part. Two concentrated forces are applied to the structure. For convenience, a $2 \times n$ description matrix $\mathbf{D}$ is introduced to describe the state of the internal load of each component. When $F_0$ increases to 15.33 kN, Member 1 fails. The corresponding description matrix $\mathbf{D}$ is

$$\begin{bmatrix} \text{member} \\ \nu_i \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 1 & 0.08717 & 0.97729 & 0.61528 & 0.58313 & 0.32518 & 0.23976 & 0.23976 & 0.45986 & 0.33908 \end{bmatrix}$$

Member 1 is removed and the external load $R_0$ is retained, and the redistribution of internal load results in the failure of Members 3 and 5. The corresponding description matrix $\mathbf{D}$ is

$$\begin{bmatrix} \text{member} \\ \nu_i \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ - & 0.98362 & 1 & 0.79894 & 1 & 0.42873 & 0.13621 & 0.13621 & 0.60629 & 0.19264 \end{bmatrix}$$

The analysis is terminated because the structure becomes a mechanism. The bearing capacity of the intact structure is 15.33 kN.

A damaged structure is obtained by removing Member 3. Member 3 fails when $F_0$ reaches 7.68 kN. The corresponding description matrix $\mathbf{D}$ is

$$\begin{bmatrix} \text{member} \\ \nu_i \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 0.98996 & 0.39497 & - & 1 & 0.40000 & 0.11212 & 0.17070 & 0.17070 & 0.15859 & 0.24144 \end{bmatrix}$$

The structure becomes a mechanism, and bearing capacity of the damaged structure is 7.68 kN. The failure scenarios are traced in Fig. 6.

The importance coefficient of Component 3 can be calculated as

$$\gamma_3 = 1 - \frac{R_3}{R_0} = 1 - \frac{7.68}{15.33} = 0.49902$$

### Frame Component

For a frame structure, the judgment of component removal is difficult to make because the component failure criteria under the combined loadings of bending, shear, and axial compression (tension) are complicated. Besides, the presence of plastic hinges also makes it difficult to describe the failure scenarios in a simplified way as previously done for the truss structures. Therefore, the complex and trivial failure scenarios of frame structures were neglected, and load-bearing capacity was emphasized.

Generally, there are mainly two kinds of collapse modes, i.e., vertical collapse mode and lateral collapse mode. The bearing capacity of the vertical and lateral collapse mode can be evaluated by pushdown and pushover analysis, respectively. However, difficulties exist when pushdown analysis is applied to calculate the bearing capacity of the damaged and intact structure:

1. Variation in monitored point for the damaged structure: The typical model considered in pushdown analysis is a damaged structure with removal of a column. Usually, displacement of the node above the removed column (i.e., the monitored point in pushover analysis) is of concern. However, the monitored point varies with the column removal scenario. Though curves of bearing capacity versus displacement of monitored points can be obtained through pushdown analysis, those curves cannot be compared to reflect importance of the column to structure in that they represent the responses of different nodes.
2. Selection of monitored point for the intact structure: When pushdown analysis is applied to the intact structure, no node is appropriate to be selected as the monitored point because the vertical displacement of each node is almost zero.

Therefore, lateral collapse mode is considered at the current stage, and pushover analysis was implemented as a substitute for the idealized incremental method. In fact, pushover itself is an incremental method based on either incremental force or incremental displacement theory. It can also reflect the distribution of plastic hinges as well as the collapse process under a certain event. However, only the results are concerned in this study.

Different from truss structures, in which the incremental analysis terminates upon the condition when the truss becomes a mechanism, pushover analysis stops at the state when the monitored point reaches the target displacement. The curve of base shear force and displacement can be obtained through pushover analysis. Base shear force typically reaches its maximal value before the structure is pushed to the large target displacement set previously, and the maximal base shear force is regarded as the load-bearing capacity of the structure. Applying the same pushover load to the structure with and without the removal of a component, the load-bearing capacity of the damaged structure $V_i$ and intact structure $V_0$ can be obtained. Further, the importance coefficient of the removed component can be calculated. This process is illustrated in Fig. 7.

**Table 1.** Section Properties of All Trusses

| Truss | Member | Diameter (mm) | Area ($mm^2$) | $E$ ($N/mm^2$) | $F_y$ ($N/mm^2$) | $N_{max}$ ($P_{max}$) (N) |
|---|---|---|---|---|---|---|
| A | 1–10 | 12 | 113.10 | 200,000 | 240 | 27,143 |
| B | 2 and 6–10 | 12 | 113.10 | 200,000 | 240 | 27,143 |
| | 4 and 5 | 20 | 314.16 | 200,000 | 240 | 27,143 |
| | 1 and 3 | 24 | 452.39 | 200,000 | 240 | 27,143 |
| C | 1–11 | 12 | 113.10 | 200,000 | 240 | 27,143 |
| | 12 and 13 | 22 | 380.13 | 200,000 | 240 | 27,143 |
| D | 2 and 6–10 | 12 | 113.10 | 200,000 | 240 | 27,143 |
| | 4 and 5 | 20 | 314.16 | 200,000 | 240 | 27,143, |
| | 11 and 12 | 22 | 380.13 | 200,000 | 240 | 27,143 |
| | 1 and 3 | 24 | 452.39 | 200,000 | 240 | 27,143 |

Note: All components have a solid circular section.

### Robustness Index

The controversial term robustness and vulnerability are used differently by different writers and there is no general agreement today as to its precise meaning. The present robustness assessment theory is based on vulnerability, whose meaning is also different from the

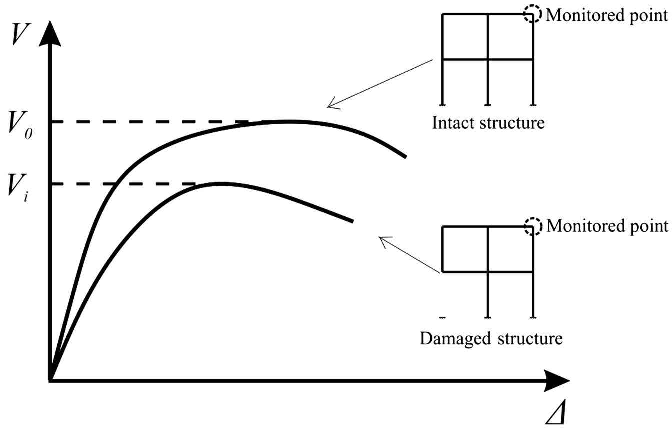


**Fig. 7.** Pushover analysis of the intact and damaged structure

mainstream comprehension. In order to demonstrate clearly, it is necessary that the meaning of controversial concepts be reemphasized. Two frameworks for robustness assessment are introduced in the following sections.

## Framework for Robustness Assessment

To distinguish the meaning of each term, a typical probabilistic framework developed by Starossek and Haberland (2010) and a semideterministic framework adopted in the present paper are depicted in Fig. 8.

In the probabilistic framework, exposure results from the abnormal events that possibly affect a structure during construction and lifetime and are not considered in ordinary structure design. The term event in the semideterministic framework is the short form for abnormal event, which is similar to the meaning of exposure. The present research is focused on illustrating a new theory of robustness assessment rather than conducting finite-element analysis for an accurate collapse simulation. Therefore, the event considered here means a distribution of specific load and is achieved by overload due to an incremental increase in the applied load.

Robustness is defined as the insensitivity of a structure to initial damage, and in a similar way, collapse resistance is defined as the

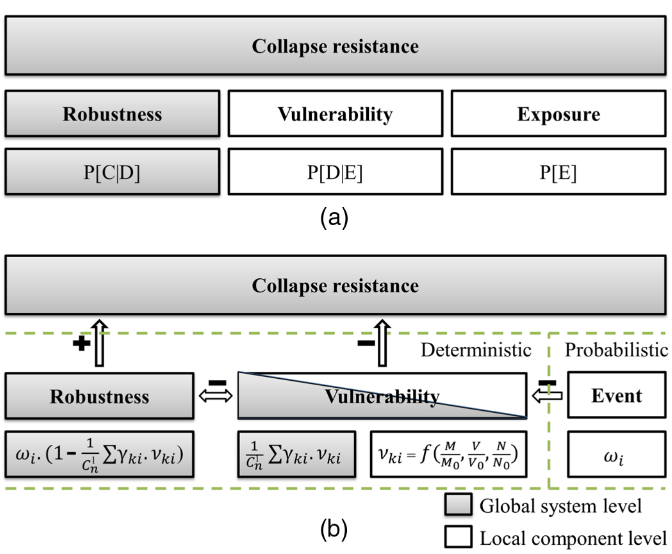


**Fig. 8.** Two frameworks for robustness assessment: (a) probabilistic framework; (b) semideterministic framework

insensitivity of a structure to abnormal events (Starossek 2006). However, it is emphasized that in the semideterministic framework, robust is not an intrinsic static property of the intact structure, but a dynamic property relevant to topology and extent of vulnerability and changes throughout the collapse process. Robustness in both frameworks is related to global system behavior.

Vulnerability is defined as susceptibility of a structure to suffer initial damage when affected by abnormal events (Starossek et al. 2011). In the context of probabilistic framework, vulnerability accounts for the direct consequences of an abnormal event, which are related to the local component behavior. However, in the context of semideterministic framework, vulnerability is related to component behavior as well as structural behavior. Vulnerability at the local component behavior is indicated by a vulnerability coefficient $\nu_{ki}$, which is a function of the internal forces and bearing capacity of the component, while vulnerability at the global system level is defined as the total contribution of component vulnerability.

## Robustness under a Single Event

According to the semideterministic framework, structural vulnerability is viewed as the antonym of structural robustness from the perspective of global system level. If a structure is deemed as an assembly of discrete components and the structural vulnerability as a contribution of component vulnerability with important components dedicating more, then the vulnerability can be defined as the sum of component vulnerability weighted by the corresponding importance coefficient, as calculated by

$$VI_i = \frac{1}{C_n^1}\sum_{k=1}^{n}\gamma_{ki}\cdot\upsilon_{ki} \tag{8}$$

where $C_n^1$ = number of all possibilities when 1 out of $n$ components is removed, and its reciprocal, i.e., $1/C_n^1$, is a factor to control $VI_i$ to take the value between 0 and 1. Variables $\gamma_{ki}$ and $\nu_{ki}$ denote the importance coefficient and vulnerability coefficient of component $k$ under event $i$, respectively.

As the opposite of structural vulnerability, structural robustness under event $i$ is defined as Eq. (9)

$$RI_i = 1 - \frac{1}{C_n^1}\sum_{k=1}^{n}\gamma_{ki}.\upsilon_{ki} \tag{9}$$

In Eqs. (8) and (9), only one component is expected to fail at the LDS. If an extremely abnormal event leads to initial damage of $m$ arbitrary components, i.e., a component group, simultaneously, then the structural vulnerability and robustness should be revised as Eqs. (10) and (11), respectively

$$VI_i = \frac{1}{C_n^m}\sum_{g=1}^{C_n^m}\gamma_{gi}.\upsilon_{gi} \tag{10}$$

$$RI_i = 1 - \frac{1}{C_n^m}\sum_{g=1}^{C_n^m}\gamma_{gi}\cdot\upsilon_{gi} \tag{11}$$

where $C_n^m$ = number of all possibilities when $m$ out of $n$ components are removed; and $\gamma_{gi}$ and $\nu_{gi}$ = importance coefficient and vulnerability coefficient of component group $g$ under event $i$, respectively. Given that initial failure of a component group is of low probability and the calculation of its relevant coefficients is very complex, it is not considered in this paper and will be introduced in detail in follow-up studies. Both $VI_i$ and $RI_i$ take the value between 0 and 1, with larger $VI_i(RI_i)$ values indicating the structure being more vulnerable (robust).



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### *Robustness under Different Events*

Given that structural robustness is a property reflected by a structure under the effects of certain events, any quantification of structural robustness is meaningless unless the events from which the structure suffers are specified. A metaphor of *Angry Birds*, a popular mobile game, might provide a vivid illustration of the relationship between robustness and events. The type of bird utilized to shoot the target house, the flight path of the bird, and the intensity of the strike are three parameters that define an event. Changing any one of the parameters will trigger a different event to which the target house will respond differently. For engineering structures, a certain event is defined when a specific load distribution or displacement is exerted on the structure. The event could either be a natural disaster, such as an earthquake, or a terrorist attack like the one on September 11, 2001. During a severe earthquake, seismic forces transformed into inertia forces distributed on the structure and vary with the earthquake wave during the period; this condition may result in the initial failure of several components in the different part of a structure. While the impact load destroys the affected components within an extremely short time, the components that initially failed are concentrated in the affected part of the structure. The failure scenarios and robustness in earthquake and impact events vary because of the disparities in the initial damage and duration of impulse.

Eq. (9) provides the robustness index under a certain event. However, how do researchers assess the robustness of a structure by considering earthquake and impact events simultaneously? Modeling structural boundaries, such as uncertain events, can be difficult; however, supposing that the probability of occurrence of event $i$ and event $j$ can be estimated by prevention agencies, then the robustness under the two events is provided by

$$RI = \omega_i.\left(1 - \frac{1}{n}\sum_{k=1}^{n}\gamma_{ki}.\upsilon_{ki}\right) + \omega_j.\left(1 - \frac{1}{n}\sum_{k=1}^{n}\gamma_{kj}.\upsilon_{kj}\right) \quad (12)$$

where $\omega_i$ and $\omega_j$ = probability of occurrence of event $i$ and event $j$, respectively.

Certain discreet events can be regarded as discrete random variables with probability of occurrence. A more general form of Eq. (12) is provided by

$$RI = \sum_i \omega_i.\left(1 - \frac{1}{n}\sum_{k=1}^{n}\gamma_{ki}\cdot\upsilon_{ki}\right) \quad (13)$$

If all events the structure suffers from during its lifetime can be predicted, then these events become continuous random variables that are described by the occurrence probability density function $\omega(x)$, which should satisfy

$$\lim_{i\to\infty}\sum_i \omega_i = \int_{-\infty}^{\infty}\omega(x)dx = 1 \quad (14)$$

Therefore, robustness under continuous events is calculated as

$$RI = \int_{-\infty}^{\infty}\omega(x)\cdot\left(1 - \frac{1}{n}\sum_{k=1}^{n}\gamma_{kx}\cdot\upsilon_{kx}\right)dx \qquad \int_{-\infty}^{\infty}\omega(x)dx = 1 \quad (15)$$

Due to the complexity in modeling uncertainty, it is difficult to find out an exact occurrence probability density function for abnormal event like an earthquake. Thus, the events discussed in this paper are simplified into specific load distribution, and robustness is evaluated under different discrete events.

## Numerical Examples

To illustrate the calculation of a robustness index and the relativity of robustness with respect to different events, case studies on a RC frame and three truss structures are presented as follows, for the purpose of illustrating the calculation.

### *RC Frame Structure*

For the planar frame given in Fig. 9, all the beams share the same section details, as do the columns. Two incremental concentrated loads of $0.5F$ and $F$ are assigned to the left-side nodes on the first and second floors, respectively. The value of distributed loads on the first and second floors is 10 kN/m.

Pushover analysis with a target displacement of 400 mm was applied to the intact and 10 damaged structures. Such analysis was also the basis for the attainment of the shear force versus

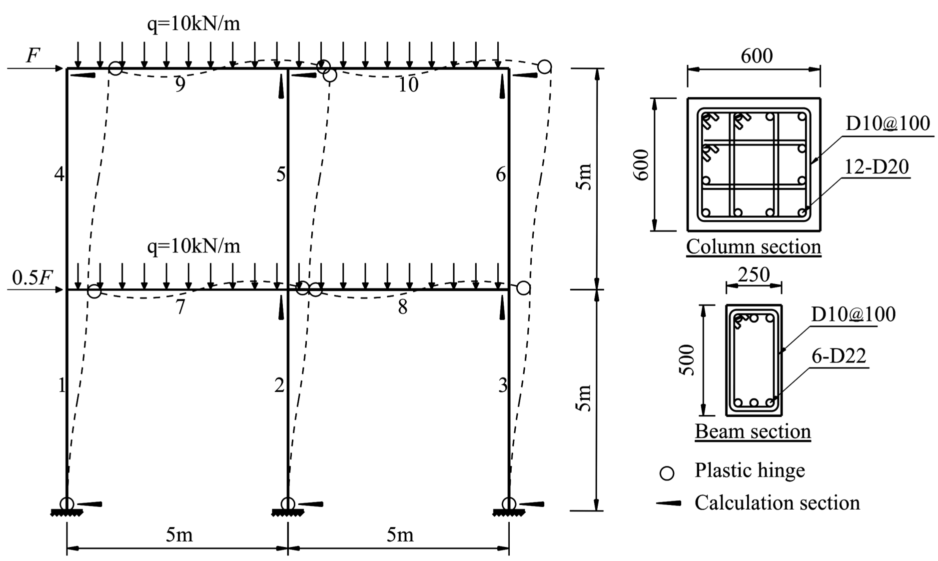


**Fig. 9.** Pushover load case and section details of a RC frame



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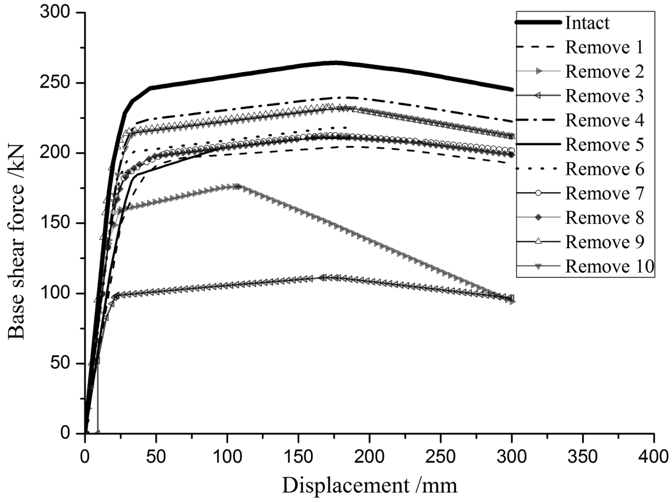


**Fig. 10.** Pushover curve of the intact and damaged structures

displacement curve (Fig. 10). The maximal base shear forces are listed in Table 2 and the importance coefficient of each component is calculated. When the intact frame is pushed to a displacement of 175.43 mm, the base shear force reaches its maximal value of 264.23 kN, i.e., the stage for vulnerability assessment of components, from which the maximum value of axial force, shear force, and bending moment of all the components can be calculated. According to the section details, the ultimate pure axial compression (tension) capacity, ultimate pure shear capacity, and ultimate bending moment capacity of beams and columns can also be calculated. Table 2 summarizes the calculation results of $N(P)$, $M$, $V$, $N_0(P_0)$, $M_0$, and $V_0$ and the vulnerability coefficient of each component.

Table 2 shows that the importance coefficients of the columns of the first floor are larger than those of the second floor. The beams are less important than any of the columns except Column 4. The vulnerability coefficient of Component 3 is equal to 1, which means that this component failed. Component 4 has a vulnerability coefficient of 0.72367; this value indicates that this component did not fail and could still sustain additional force because Component 4 was located in the second floor and underwent tension rather than compression. The vulnerability coefficients of the other components are close to 1. This condition means that these components would fail under a slightly larger load. Unlike the condition in the truss, the formation of plastic hinges within the frame can redistribute the internal forces within the frame, and can thus delay the emergence of a mechanism. The relatively even internal force distribution allows each component to fully utilize the bearing capacity of its section. The robustness index for the RC frame is 0.78682 with respect to the base shear force of 264.23 kN at a displacement of 175.43 mm.

## Truss Structures under Three Different Events

Three identical 11-bar trusses, all denoted as Truss A, were subjected to the effect of three different events called Event A, Event B, and Event C, as shown in Fig. 11. It was assumed that all truss

**Table 2.** Calculation Results of the RC Frame

| Member | $N(P)$ (kN) | $M$ (kN · m) | $V$ (kN) | $N_0(P_0)$ (kN) | $M_0$ (kN · m) | $V_0$ (kN) | $\upsilon$ | $F_i$ (kN) | $\gamma$ |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 46.03 | 227.22 | 79.79 | 1,005.31 | 261.38 | 4,515.50 | 0.91517 | 204.42 | 0.22635 |
| 2 | −167.83 | 287.71 | 85.50 | 5,837.94 | 261.38 | 4,515.50 | 0.98105 | 176.03 | 0.33382 |
| 3 | −233.02 | 312.55 | 101.93 | 5,837.94 | 261.38 | 4,515.50 | 1.0023 | 111.19 | 0.57920 |
| 4 | 50.42 | 176.04 | 37.21 | 1,005.31 | 261.38 | 4,515.50 | 0.72367 | 239.52 | 0.09353 |
| 5 | −62.75 | 266.53 | 102.32 | 5,837.94 | 261.38 | 4,515.50 | 0.98362 | 210.94 | 0.20168 |
| 6 | −75.13 | 185.50 | 38.21 | 5,837.94 | 261.38 | 4,515.50 | 0.82725 | 218.14 | 0.17444 |
| 7 | −44.58 | 202.91 | 99.58 | 2,403.97 | 190.67 | 1,590.19 | 0.98727 | 212.65 | 0.19521 |
| 8 | −62.11 | 202.63 | 99.56 | 2,403.97 | 190.67 | 1,590.19 | 0.96959 | 211.36 | 0.20008 |
| 9 | −138.01 | 186.48 | 95.16 | 2,403.97 | 190.67 | 1,590.19 | 0.85844 | 232.55 | 0.11989 |
| 10 | −36.92 | 185.50 | 75.77 | 2,403.97 | 190.67 | 1,590.19 | 0.95002 | 231.79 | 0.12275 |

Note: For axial force, the positive and negative values mean that the component is under tension and compression, respectively. The absolute value of axial force is used when substituted in Eq. (6). $Fi$ denotes the bearing capacity of the damaged structure after the removal of component $i$.

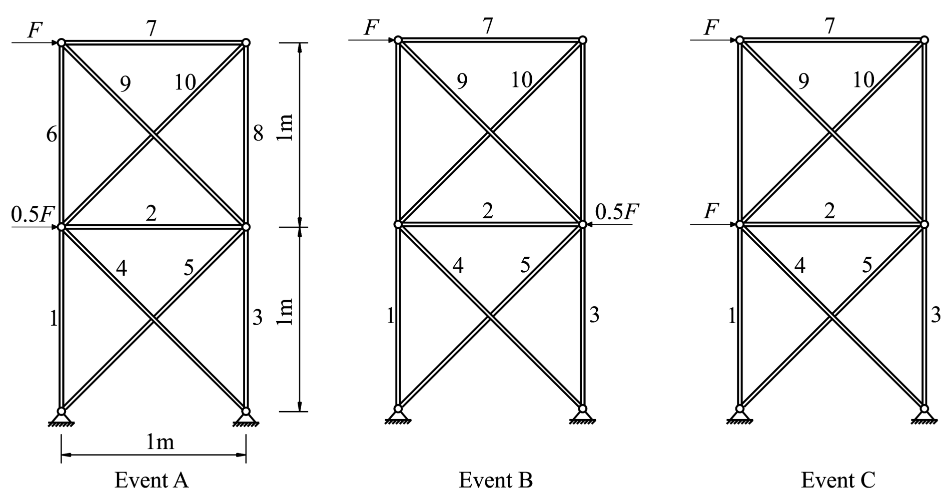


**Fig. 11.** Truss A under three different events

Table 3. Calculation of Coefficients under Three Events

| Component | Event A $\gamma$ | Event A $\upsilon$ | Event B $\gamma$ | Event B $\upsilon$ | Event C $\gamma$ | Event C $\upsilon$ |
|---|---|---|---|---|---|---|
| 1 | 0.49934 | 1.00000 | 0.49196 | 1.00000 | 0.51765 | 1.00000 |
| 2 | 0.04422 | 0.08717 | 0.06058 | 0.12146 | 0.08426 | 0.17342 |
| 3 | 0.49934 | 0.97729 | 0.49196 | 0.96835 | 0.51765 | 0.95470 |
| 4 | 0.16556 | 0.61528 | 0.15327 | 0.22207 | 0.31787 | 0.72312 |
| 5 | 0.16556 | 0.58313 | 0.15327 | 0.25593 | 0.31787 | 0.65906 |
| 6 | 0.03621 | 0.32518 | 0.10190 | 0.45318 | 0.03266 | 0.29259 |
| 7 | 0.00495 | 0.23976 | 0.10190 | 0.33416 | −0.02320 | 0.19608 |
| 8 | 0.00495 | 0.23976 | 0.10190 | 0.33416 | −0.02320 | 0.19608 |
| 9 | 0.03621 | 0.45986 | 0.10190 | 0.64089 | 0.03266 | 0.41379 |
| 10 | 0.00495 | 0.33908 | 0.00688 | 0.47258 | −0.02320 | 0.27729 |

components are made of brittle materials. The section properties are presented in Table 1. The calculation of coefficients in Table 3 and distribution of coefficients in Fig. 12 indicate the following:

- The three trusses all have something in common, namely, Components 1 and 3 are most important, followed by Components 4 and 5. This result is attributed to the fact that both upper lateral forces, 1.5$F$ in total, are transferred to the foundation through the columns and braces. When one of these four components fails, the other three components have to bear the load originally carried by the failed one. This condition always triggers progressive collapse. Components 7, 8, and 10 are the least important ones because the failure of any one of them would only result in the redistribution of the lateral load of the second floor, $F$ in total, and has minimal influence on the bearing capacity reduction of the intact truss;
- For Event B, the failure of any one component among Components 6, 7, 8, and 9 will cause the failure of the braces of the second floor, and the second floor then becomes a mechanism. Therefore, compared with Event A, the importance of Components 6, 7, 8, and 9 are improved. The importance coefficients of Components 7, 8, and 10 are no longer equal, which is mainly due to the change in the load distribution;
- For Event C, the importance coefficients of Components 7, 8, and 10 being negative implies that removing any one of these components will increase the bearing capacity of the truss; and
- Based on the premise that the section properties of all components are similar, the distribution of vulnerability reflects the internal force distribution at the stage when the bearing capacity of the truss begins to decline.

The robustness of the three trusses cannot be compared because each truss is under the effect of a different event. An event can change the internal force distribution and failure scenarios of a structure, and can thus change the distribution of both importance coefficients and vulnerability coefficients.

## Methods to Upgrade Robustness

The four trusses shown in Fig. 13 are different but under the same load distribution and section properties as the four trusses presented in Table 1. The robustness of these trusses is discussed subsequently from the perspective of failure scenarios, failure mode, and distribution of coefficients (as shown in Fig. 14).

For Truss A, Component 1 failed when $F$ increased to 15.33 kN, followed by the failure of Components 3 and 5 because of the load redistribution. The structure became a mechanism and collapsed. According to Table 4, Components 1 and 3 are important to the structure and are vulnerable in the meantime, i.e., the key elements mentioned previously. The failure of Component 1 led to the failure

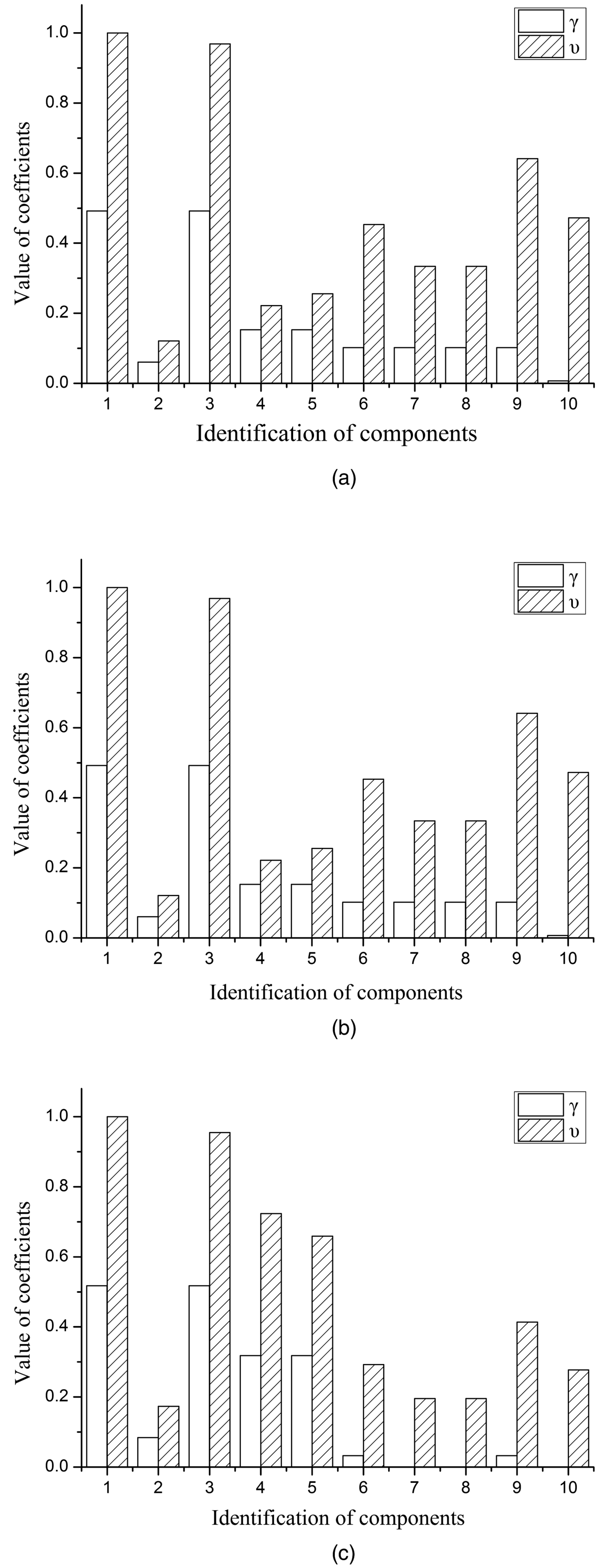


Fig. 12. Distribution of coefficients: (a) Event A; (b) Event B; (c) Event C

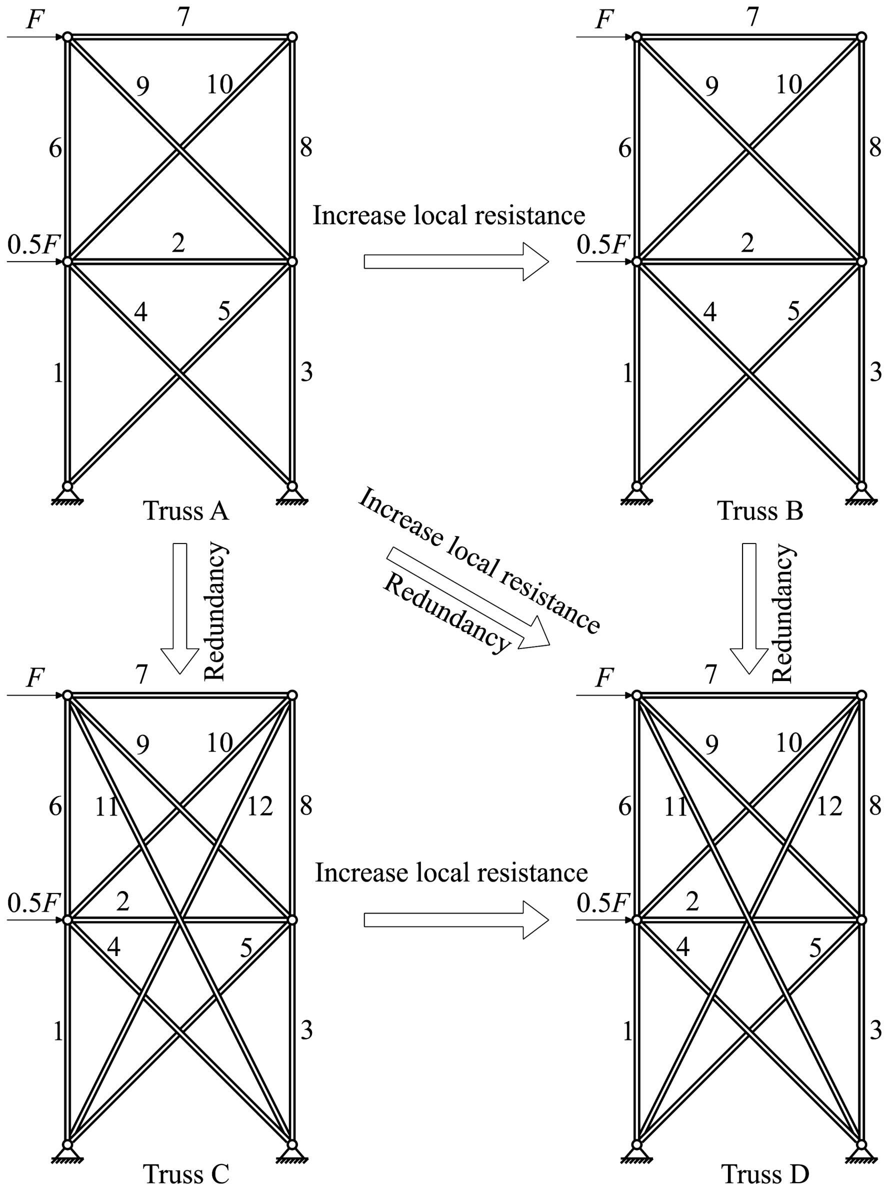


**Fig. 13.** Methods to improve structural robustness

of the other components on the first floor and triggered the overall collapse of the structure, with the second floor undamaged. This failure mode is unfavorable. Therefore, how to improve the structural robustness under a certain event? Two types of methods are considered to improve structural robustness, namely, increasing the local resistance (Starossek and Haberland 2010) and redundancy (Kanno and Ben-Haim 2011).

For Truss B, Components 1, 3, 4, and 5 are enhanced to different degrees. The first failure occurred in Component 9 when $F$ increased to 33.53 kN; this failure triggered the subsequent failure of Components 7, 8, and 10, which in turn resulted in the formation of a mechanism on the second floor. The analysis ended up with the failure of the second floor, with the first floor being intact (a desirable failure mode). Compared with those in Truss A, the importance coefficients of the first floor did not change significantly, whereas the importance coefficients of the second floor increased because the failure scenarios were concentrated on the second floor.

For Truss C, two enhanced diagonal components (i.e., Components 11 and 12) that brace both the first and second floors are added. Component 1 failed when $F$ increased to 20.29 kN, and no component failed due to the load redistribution; when $F$ increased to 38.13 kN, Component 6 failed, followed by the successive failure of Components 7 and 8. The structure finally became a mechanism. Damage existed in the two floors. Compared with Truss A, the addition of Components 11 and 12 in Truss C provides alternative load paths for the structure and thus reduces the importance and vulnerability coefficients of components of the first floor. Besides, the importance coefficients of Components 7, 8, and 10 are no longer equal as a result of the change in the form and connectivity of the structure.

Truss D exhibited mixed behavior of Truss B and Truss C. Both local resistance and redundancy increased. Component 6 failed when $F$ reached the value of 32.61 kN; force redistribution resulted in the failure of Component 8, then Components 9 and 10, and finally Components 11 and 12 in succession. Similar to the failure mode of Truss B, the first floor remained undamaged when the analysis was terminated. The importance and vulnerability coefficients decreased, similar to that of Truss C.

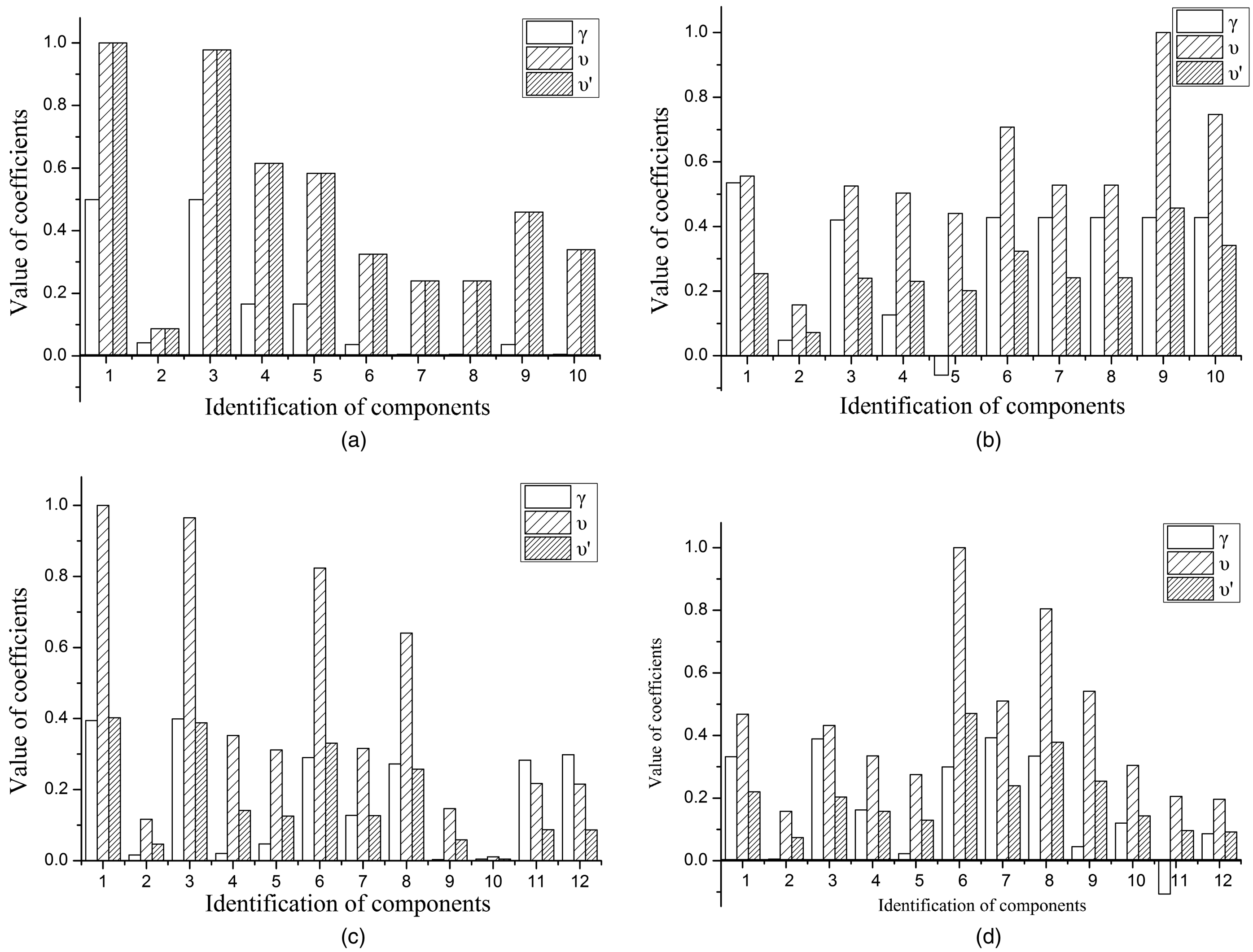


**Fig. 14.** Distribution of coefficients: (a) Truss A; (b) Truss B; (c) Truss C; (d) Truss D

**Table 4.** Calculation of Coefficients of the Four Trusses

| | Truss A | | | Truss B | | | Truss C | | | Truss D | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $F$ (kN) | 15.33 | 15.33 | $F$ (kN) | 33.53 | 15.33 | $F$ (kN) | 38.13 | 15.33 | $F$ (kN) | 32.61 | 15.33 |
| Component | $\gamma$ | $\upsilon$ | $\upsilon'$ | $\gamma$ | $\upsilon$ | $\upsilon'$ | $\gamma$ | $\upsilon$ | $\upsilon'$ | $\gamma$ | $\upsilon$ | $\upsilon'$ |
| 1 | 0.49934 | 1.00000 | 1.00000 | 0.53501 | 0.55589 | 0.25425 | 0.39488 | 1.00000 | 0.40216 | 0.33220 | 0.46853 | 0.22031 |
| 2 | 0.04222 | 0.08717 | 0.08717 | 0.04798 | 0.15724 | 0.07192 | 0.01611 | 0.11671 | 0.04694 | 0.00526 | 0.15750 | 0.07406 |
| 3 | 0.49934 | 0.97729 | 0.97729 | 0.42022 | 0.52489 | 0.24007 | 0.39888 | 0.96485 | 0.38803 | 0.38965 | 0.43185 | 0.20307 |
| 4 | 0.16556 | 0.61528 | 0.61528 | 0.12646 | 0.50318 | 0.23014 | 0.02043 | 0.35242 | 0.14173 | 0.16232 | 0.33534 | 0.15768 |
| 5 | 0.16556 | 0.58313 | 0.58313 | −0.06015 | 0.44008 | 0.20128 | 0.04722 | 0.31185 | 0.12541 | 0.02275 | 0.27493 | 0.12928 |
| 6 | 0.03621 | 0.32518 | 0.32518 | 0.42752 | 0.70714 | 0.32343 | 0.28984 | 0.82337 | 0.33113 | 0.29997 | 1.00000 | 0.47022 |
| 7 | 0.00495 | 0.23976 | 0.23976 | 0.42752 | 0.52804 | 0.24151 | 0.12728 | 0.31642 | 0.12725 | 0.39298 | 0.51018 | 0.23990 |
| 8 | 0.00495 | 0.23976 | 0.23976 | 0.42752 | 0.52804 | 0.24151 | 0.27238 | 0.64054 | 0.25760 | 0.33418 | 0.80490 | 0.37848 |
| 9 | 0.03621 | 0.45986 | 0.45986 | 0.42752 | 1.00000 | 0.45738 | 0.00328 | 0.14679 | 0.05903 | 0.04496 | 0.54097 | 0.25438 |
| 10 | 0.00495 | 0.33908 | 0.33908 | 0.42752 | 0.74679 | 0.34156 | 0.00454 | 0.01087 | 0.00437 | 0.12003 | 0.30468 | 0.14327 |
| 11 | — | — | — | — | — | — | 0.28291 | 0.21773 | 0.08756 | −0.10623 | 0.20540 | 0.09658 |
| 12 | — | — | — | — | — | — | 0.29852 | 0.21563 | 0.08672 | 0.08640 | 0.19607 | 0.09220 |

As stated previously, robustness herein is a dynamic property relevant to external event under which the structure will be assessed. Assuming that the four trusses are subjected to the same load of 15.33 kN (i.e., ultimate bearing capacity of Truss A), according to the coefficients in Table 4, the relative robustness index for Trusses A, B, C, and D is 0.87779, 0.90563, 0.89509, and 0.95255, respectively. This result matches well with the assumption set in the design stage of numerical cases. The robustness index of Trusses A, B, C, and D corresponding to each ultimate bearing capacity is 0.87779, 0.79367, 0.73913, and 0.89546, respectively. This finding seems to contradict this paper's assumption because the robustness index of Truss B is smaller than that of Truss A.

In fact, this comparison makes no sense because the robustness indexes are obtained under different events, i.e., different ultimate bearing capacities.

The preceding measures adopted to improve structural robustness are empirical. Though robustness of the original truss has been upgraded, it is at the expense of increasing the size of the key elements and number of elements, which is not economical. Despite all this, the two methods have proved the correctness and reasonableness of the present robustness theory. An accurate algorithm should be developed to optimize the structural robustness in the future research.

## Conclusions

- Structural robustness is not only a property of the form and connectivity of a structure but is also relevant to uncertain abnormal events from which the structure will suffer.
- Robustness changes throughout the entire collapse process. The locally damaged stage is closest to the original stage; the bearing capacity typically begins to decline during this stage. Therefore, this stage is selected as the stage for the assessment of structural vulnerability in most cases.
- The importance coefficient bridges the gap between local components and the global structure and reflects the failure scenarios and topology of the structure. This coefficient is utilized as a weight coefficient of the vulnerability coefficient to obtain structural vulnerability, which is considered as the opposite of structural robustness.
- Both importance and vulnerability coefficients are relevant to external events. A change in structural topology or a variation in events will influence the distribution of these two coefficients among components. The two coefficients should be calculated under the same specified event to obtain the corresponding robustness.
- To evaluate structural robustness under certain events, occurrence probability is introduced to model the uncertainty of abnormal discrete events. If all events the structure will suffer from during its lifetime can be predicted, then these events can be regarded as continuous random variables described by the occurrence probability density function.
- Increasing the local resistance of key elements and redundancy of a structure are two effective methods to improve structural robustness. An algorithm should be further developed for accurate optimization.

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